International Journal of Recent Technology and Engineering (IJRTE)
ISSN: 2277-3878 (Online), Volume-8 Issue-6, March 2020

Numerical Modeling Methods for Large Open Loop Multibody System

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Abstract*: *A pendulum's motion was stated to be as a way to illustrate the movement of human body in the studies of multibody system. Therefore, a comparison between the two numerical models in multibody systems were implemented on the articulated pendulums of different sizes. The two numerical models were known as the augmented Lagrangian formulation and fully recursive method. In order to identify the difference performance of the numerical models, various size of articulated pendulums has been tested which are 2, 4, 8, 16, 20 and 40 pendulums. Differential equations developed from both models were solved by using Runge-Kutta 4 and 5. Both models were coded in Matlab and have been optimized in order to ensure only related routine were considered. The performance was evaluated based on the computing time with constant relative and absolute tolerance in Runge-Kutta solver which is 0.01 s. All pendulums were assumed to have the same weight, angle and length. As for the results, the augmented Lagrangian formulation solved the differential equations faster than the fully recursive method when tested up to 20 pendulums. However, fully recursive method started to solve the differential equations faster than the augmented Lagrangian method when it need to deal with a very large system such as 40 pendulums and above. Thus, it can be concluded that the suitable method to solve the small, open loop system such as articulated pendulums is augmented Lagrangian method while for a very large system, the fully recursive method will be more efficient.

Keywords* : *Augmented Lagrangian, Fully Recursive, Multibody Dynamics.

Manuscript received on February 10, 2020.
Revised Manuscript received on February 20, 2020.
Manuscript published on March 30, 2020.
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I. INTRODUCTION

A simple pendulum [1,2] usually will be swinging from its equilibrium position once been applied an external force. In multibody system, the pendulum can be implement to model for the motion of human body. Therefore, it is important to derive and solve the equations in order to track the motion as well as emphasizing on the initial condition's dependency [3]. The human body can be represented by articulated pendulums linked together through joints. The mass, m of each bodies may be presumed to be equally distributed with length, l. The alignment of whole bodies system is derived by angle, θ_n between bodies and the vertical axes in which represents the generalized coordinates as shown in the Fig. 1.

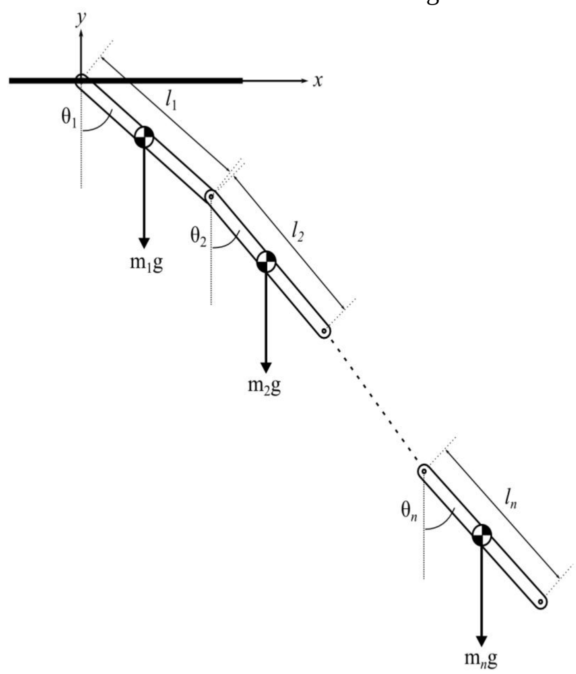

Fig. 1. Configuration of n^th articulated pendulums

In these case, the equations of motion for two multibody formulations known as the augmented Lagrangian [4],[5] and fully recursive [6]-[8] will be derived. However, the differential equations need to be defined before resolve for the equations of motion by applying the numerical time integration method. The equations of motion for the augmented Lagrangian formulation can be shown as independently redundant coordinates' set [9]. Hence, it is important to define the relation between the coordinates by the kinematic of the constraint equations.

In contrary, the elimination of constraint forces can be done by the application of fully recursive formulation as it will be expressed in term of degrees of freedom (DOF) [10]. These degrees of freedom may include few variables as the joint coordinates to reduce number of equations and increased the complexity and non-linearity of the equations.

Generally, it is compulsory to determine the kinematics of the system before deriving the equations of motion. Therefore, as part of the kinematics' definition, a global and topological method were applied to each of the formulations [11].

II. MATHEMATICAL MODEL

In the augmented Lagrangian, the equations of motion is developed on the basis of the principles of virtual work [12]. By assuming the system is unconstrained, the dynamic equilibrium usually described as:

$$\delta W_{iner} = \delta W_{ext} \quad (1)$$

whereby δW_{iner} and δW_{ext} are the inertial and external forces of virtual work respectively. δW_{iner} typically consists of mass matrix, $\mathbf{M}$, generalized coordinates, $\ddot{\mathbf{q}}$, and vector of quadratic velocity, $\mathbf{Q}_v$ meanwhile δW_{ext} consists of generalized force vector, $\mathbf{Q}_e$ of the multibody system which then can be separately written as follows:

$$\delta W_{iner} = \delta \boldsymbol{q} \cdot (\mathbf{M}\ddot{\mathbf{q}} - \mathbf{Q}_v) \quad (2)$$

$$\delta W_{ext} = \delta \boldsymbol{q} \cdot \mathbf{Q}_e \quad (3)$$

By equalizing both equations (2) and (3), a new equation will form:

$$\delta \boldsymbol{q} \cdot (\mathbf{M}\ddot{\mathbf{q}} - \mathbf{Q}_v - \mathbf{Q}_e) = \mathbf{0} \quad (4)$$

However, the relations between the generalized coordinates and constraint equations need to be justify [13]. Hence, it is compulsory to satisfy the following equation:

$$\boldsymbol{\Phi}(\mathbf{q}, t) = \mathbf{0} \quad (5)$$

It is then can simply be written as follows:

$$\mathbf{M}\ddot{\mathbf{q}} - \mathbf{Q}_e - \mathbf{Q}_v + \boldsymbol{\Phi}_{\mathbf{q}}^{T}\boldsymbol{\lambda} = \mathbf{0} \quad (6)$$

whereby $\boldsymbol{\Phi}_{\mathbf{q}}$ is the Jacobian matrix of the constraints and $\boldsymbol{\lambda}$ is a set of Lagrange multipliers. Therefore, by considering equations (5) and (6), the equations of motion may be transform in matrix such as below:

$$\begin{bmatrix} \mathbf{M} & \boldsymbol{\Phi}_{\mathbf{q}}^{T} \\ \boldsymbol{\Phi}_{\mathbf{q}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_e + \mathbf{Q}_v \\ \mathbf{Q}_c \end{bmatrix} \quad (7)$$

Equation (7) is an algebraic equations' system that can be used to solve the acceleration vector, $\ddot{\mathbf{q}}$. Full derivation for augmented Lagrangian formulation has been described structurally in [14] and [15].

In recursive formulation, the algorithm basically need the application of forward and backward motion's approach as shown in Fig. 2.

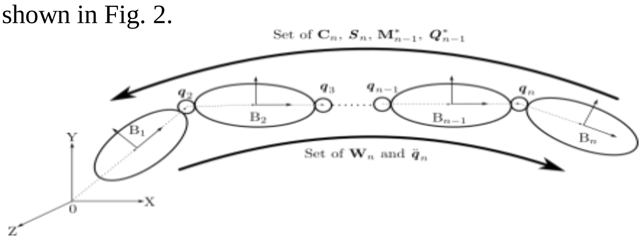

Fig. 2. Backward and forward movement approach

Any movement between the neighboring bodies and constraints by relative are significant to form matrices for the entire system and solve for their equations of motion. In simply, the derivation of the algorithm can be made as the following sequence:

i. Determine the initial state of relative coordinate, $\boldsymbol{q}(t)$ and velocities, $\dot{\boldsymbol{q}}(t)$ at t_0.

ii. Compute system's orientation and positions for $i=1$ to n_B recursively.

iii. Compute matrices of $\boldsymbol{C}_n$, $\boldsymbol{S}_n$ and updated matrices of $\boldsymbol{M}^*_{n-1}$ and $\boldsymbol{Q}^*_{n-1}$ for every bodies in backward from body B_n to B_1.

iv. Subsequently, compute an acceleration state, $\mathbf{W}_n$ and joint acceleration, $\ddot{\boldsymbol{q}}_n$ by forward method from body B_1 to B_n.

From (iv), the integration of time can be used to determine the relative coordinates. By deriving the preceding adjacent body, B_{n-1} in terms of position, $\boldsymbol{r}_{n-1}$ and velocity, $\boldsymbol{v}_{n-1}$, hence the position, $\boldsymbol{r}_n$ and velocity, $\boldsymbol{v}_n$ of the body, B_n also may be acquires. Full derivation of fully recursive formulation has been described comprehensively in [14], [15] and [16].

III. RESULTS AND DISCUSSIONS

In this studies, various size of articulated pendulums has been chosen to study for the numerical analysis and mathematical modeling for the two mentioned formulations. The ode45 solver has been used as a part to solve for the numerical analysis [17]. Runge-Kutta method of 4 and 5 order was used to integrate time of the equations of motion by manipulating step size and fixed tolerance at 0.01 s [18]. The analysing process begin with recording the computational time at maximum solution time, t_{max} is 5 s for both formulations as in Table I.

Table- I: Computing time of each step size for respective number of pendulums, N at maximum simulation time, t_{max}=5 s and tolerance setting at 0.01.

No. of pendulums, N	Step size\ Formulations	0.1	0.01	0.001	0.0001
2	Augmented	0.7944117	0.6426967	0.7765793	2.4621833
	Recursive	0.7810847	0.774487	0.9502893	2.4636347
4	Augmented	0.9611913	1.0341363	1.0082683	2.6025293
	Recursive	1.3397943	1.233603	1.496162	3.1050643
8	Augmented	1.4988773	1.394732	1.581287	3.5003467
	Recursive	2.624756	2.4609253	2.9304427	4.366564
16	Augmented	4.776052	4.2883407	4.8907053	7.0527577
	Recursive	5.6107193	5.590695	5.888236	7.720457
20	Augmented	6.7254363	6.4991763	6.696233	10.030193
	Recursive	7.927445	8.353048	8.0224817	9.686146
40	Augmented	29.159576	28.526508	31.478935	32.6064423
	Recursive	23.3639643	22.961879	23.946438	26.4761887

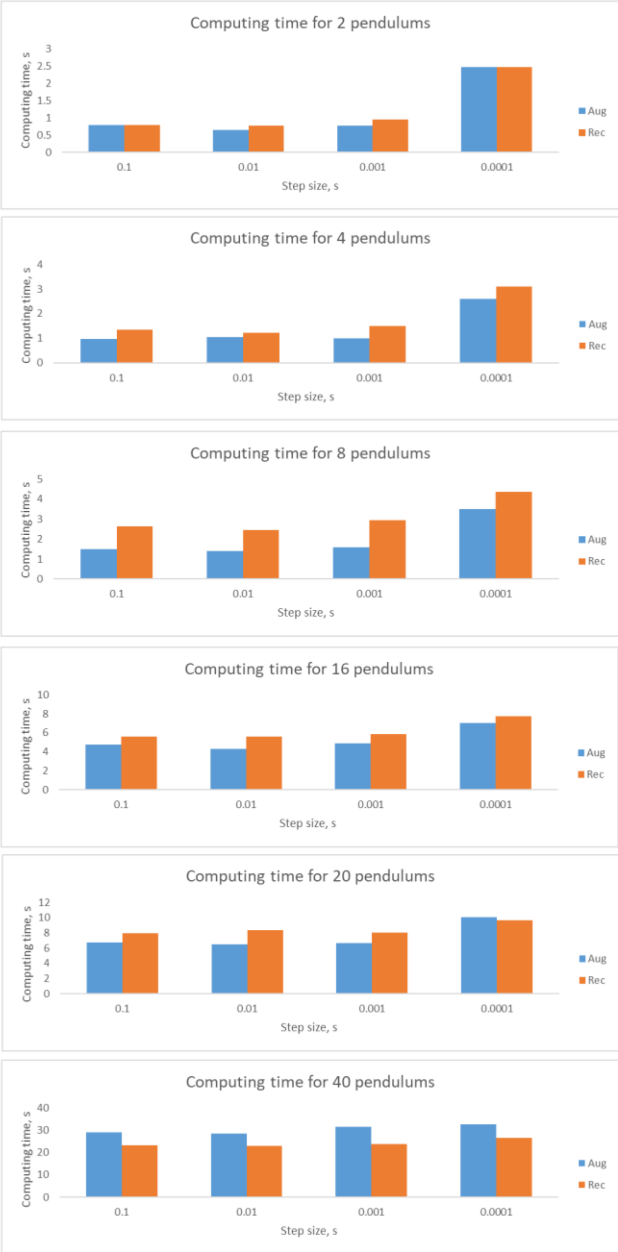

Fig. 3. Computing time for difference size of pendulums

Generally, it is found that as the articulated pendulums became larger and longer, both formulations will consume more time to compute within the tightening of the step size. Pendulums with smaller tails such that number of pendulums, N are two, four, eight, sixteen and twenty shows slightly difference in computing time between the augmented Lagrangian and fully recursive methods. Also, for those smaller number of pendulums, it can be seen that the augmented Lagrangian solve the system faster than the fully recursive except at the step size 0.0001 s for N=20. However, as the articulated pendulums up to forty bodies, a significance result between these two formulisms can be seen as the differences between augmented Lagrangian and fully recursive show huge gaps in computing time. The time taken by fully recursive formulism are now computes faster than the augmented Lagrangian formulations. Eventhough by theoretically, the recursive should be more efficient to solve for the multibody system [16] yet from the results, it can be presumed that the fully recursive computes better than the augmented Lagrangian when dealing with large number of bodies and more complex system.

IV. CONCLUSION

A number of articulated pendulums such that 2, 4, 8, 16, 20 and 40 are chosen in order to compare the efficiency of both formulations when dealing with smaller to larger matrices. In short, it is found that at a fixed tolerance of 0.01 s, the augmented Lagrangian formulations computes faster than the fully recursive method and both are showing insignificant difference in computing time at a smaller number of pendulums up until twenty bodies except at the step size 0.0001 s for twenty articulated pendulums. However, as the pendulums reached forty articulations, the fully recursive method showed a significance results compared to augmented Lagrangian formulation as the differences in computing time are huge between both formulisms. Therefore, it can conclude that the augmented Lagrangian requires least time to solve for the system at smaller matrices meanwhile the fully recursive works better for large matrices and complex system.

ACKNOWLEDGMENT

This work was supported by the Fundamental Research Grant Scheme (FRGS) under a grant number of FRGS/1/2018/TK03/UNIMAP/02/19 from the Ministry of Education Malaysia.

REFERENCES

1. J. Nagel, “Mathematical description of the objects,” Lect. Notes Energy, vol. 69, pp. 101–161, 2019.
2. J. C. Lacunza, “The Pendulum of Dynamic Interactions,” J. Appl. Math. Phys., vol. 03, no. 09, pp. 1186–1198, 2015.
3. Y. J. Mahbouba and M. Q. Abdullah, “Numerical Solution of Linear and Nonlinear Swinging Pendulum,” vol. 4, no. 5, pp. 102–109, 2017.
4. S. Stupkiewicz, J. Lengiewicz, and J. Korelc, “Sensitivity analysis for frictional contact problems in the augmented Lagrangian formulation,” Comput. Methods Appl. Mech. Eng., vol. 199, no. 33–36, pp. 2165–2176, 2010.
5. E. Paraskevopoulos, N. Potosakis, and S. Natsiavas, “An augmented Lagrangian formulation for the equations of motion of multibody systems subject to equality constraints,” Procedia Eng., vol. 199, pp. 747–752, 2017.
6. P. Fisette and C. Jo, “Symbolic generation of large multibody system dynamic equations using a new semi-explicit Newton / Euler recursive scheme,” vol. 66, pp. 187–199, 1996.
7. J. Schilder, M. Ellenbroek, and A. De Boer, “Recursive thoughts on the simulation of the flexible multibody dynamics of slender offshore structures,” IOP Conf. Ser. Mater. Sci. Eng., vol. 276, no. 1, 2017.
8. A. Müller, “Screw and Lie group theory in multibody kinematics: Motion representation and recursive kinematics of tree-topology systems,” Multibody Syst. Dyn., vol. 43, no. 1, pp. 37–70, 2018.
9. A. A. Shabana and J. R. Sany, “Augmented formulation for mechanical systems with non-generalized coordinates: Application to rigid body contact problems,” Nonlinear Dyn., vol. 24, no. 2, pp. 183–204, 2001.
10. S. K. Ider, “Finite-element based recursive formulation for real time dynamic simulation of flexible multibody systems,” Comput. Struct., vol. 40, no. 4, pp. 939–945, 1991.
11. J. Cuadrado and D. Dopico, “A Combined Penalty and Semi-Recursive Formulation for Closed-Loops in MBS,” Proceedings of the 11th World Congress in Mechanism and Machine Science April 1-4, 2004.
12. M. E. Baharudin, P. Korkealaakso, A. Rouvinen and A. Mikkola, “Crane Operators Training Based on the Real-Time Multibody Simulation,” Multibody Syst. Dyn, Robotics and Control, Springer, pp. 213–229, 2013.

13. T. R. Kane and D. A. Levinson, “Multibody Dynamics,” J. Appl. Mech., vol. 50, no. 4b, p. 1071, 2009.
14. A. A. Shabana, “Computational Dynamics,” John Wiley and Sons, 2010.
15. A. A. Shabana, “Dynamics of Multibody Systems,” Cambridge University Press, 2005.
16. Slaats, “Recursive Formulations in Multibody Dynamics,” Eindhoven: Technische Universiteit Eindhoven.
17. E. Emagbetere, O. O. Oluwole, and T. A. O. Salau, “Comparative Study of Matlab ODE Solvers for the Korakianitis and Shi Model,” Bull. Math. Sci. Appl., vol. 19, pp. 31–44, 2017.
18. T. Ritschel, “Numerical Methods For Solution of Differential Equations,” Department of Applied Mathematics and Computer Science, The Technical University of Denmark, 2013.

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