

Viscous Dissipation Effect on 3D Unsteady MHD Flow Through a Vertical Porous Plate with Radiation, Heat Source and Slip Boundary Conditions



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Abstract: An unsteady three dimensional MHD flow of an incompressible, viscous, electrically conducting fluid through a moving vertical porous plate with periodic Suction of infinite length in the presence of Heat source, viscous dissipation and radiation effect in slip flow regime is analyzed and the governing equations are solved by perturbation technique. The nature of various dimensionless parameters on velocity, temperature, skin friction and rate of heat transfer are analyzed with the help of graphs.

Key Words: Radiation Effect, slip flow, Heat Source, viscous dissipation

I. INTRODUCTION:

Over the past few years that more focus and attention is being shown as regards to fluid flows through porous media as research scholars have evinced keen interest and were attracted by similar flows in many fields of science and technology. With special reference to agricultural engineering, it helps to analyze the underground water resources, discharge of water in river-beds, in petroleum technology to observe the movement of natural gas, water and oil through oil reservoirs, in chemical engineering for filtration and purification processes.

Guria and Jana [4] analyzed the unsteady hydrodynamic free convective flow of an incompressible viscous fluid past a vertical porous plate with the presence of variable Suction. Bhupendra Kumar Sharma, Mamta Agarwal and Chaudhary [2] investigated the effect of radiation in three-dimensional Couette flow when a transverse sinusoidal injection velocity at the stationary plate of the channel is proposed.

Das, Mohanty, Panda and Sahoo [3] analyzed the transverse magnetic field effect on 3D couette flow of a viscous, electrically conducting, incompressible fluid between two infinite horizontal parallel porous plates.

Ahmed [1] studied MHD free and forced convection three dimensional flow of a viscous, electrically conducting fluid with mass transfer along a vertical porous plate with transverse sinusoidal suction velocity.

Sahin Ahmed [11] discussed the 3D flow of an incompressible viscous fluid over an infinite vertical porous plate with periodic suction velocity and viscous dissipative heat when the free stream velocity oscillates in time about a non-zero constant mean.

Rajput and Kumar [9] investigated the radiation effect on MHD flow past an impulsively started vertical plate with varying heat and mass transfer. Ravikumar, Raju and Raju [10] studied the hydro magnetic effects of an electrically conducting flow of a viscous incompressible fluid through a porous medium bounded between infinite vertical porous plates with periodic suction velocity at constant temperature.

Guria [6] analyzed about the nature of the flow of a viscous incompressible fluid through a vertical channel in the presence of radiation effect immersed in a porous medium.

Guria, Ghara and Jana [7] analyzed the 3D flow past a vertical porous plate with radiation effect in a porous medium subject to the periodic suction velocity. Guria [8] studied the effect of radiation and slip boundary condition on a three dimensional flow of an incompressible viscous fluid past a vertical channel.

Guria and Jana [5] investigated the radiation effect and magnetic field on three dimensional flow past a vertical porous plate subject to periodic suction velocity. The present study is aimed at formulation and analysis on the effect of three dimensional unsteady flow of a viscous, incompressible, electrically conducting fluid through an infinite vertical porous plate in the presence of heat source and viscous dissipation with slip boundary condition. The periodic suction is assumed as time dependent and perpendicular to the flow direction. This makes the flow to be three dimensional. Perturbation technique is used to calculate the main and cross flow velocity. The main flow velocity and cross flow velocity, skin friction and heat transfer at the porous plate were calculated. The results arrived are validated for vanishing slip parameter and Eckert number with the results obtained by Guria and Jana [5].

II. FLOW DESCRIPTION AND GOVERNING EQUATIONS

We have considered the three dimensional unsteady flow of an incompressible, viscous, electrically conducting fluid past a semi infinite vertical porous plate in the presence of heat source and uniform magnetic field B_0 with slip flow regime.

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Here, the x^* - axis is taken along the vertical plate, that is in the direction of the flow, y^* -axis is taken perpendicular to the plate and z^* -axis is considered normal to the x^*y^* - plane.

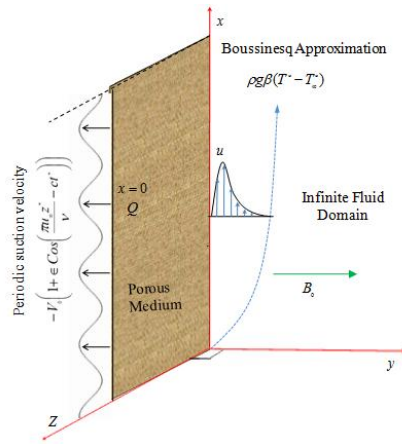


Figure 1: Physical Configuration of the problem

The upper plate is considered with constant injection $-V_0$ and the lower plate is taken with periodic suction velocity of the form

$$v^* = -V_0 \left[1 + \epsilon \cos \left(\frac{\pi u_\infty z^*}{v} - ct^* \right) \right] \quad (1)$$

where ϵ represents the amplitude of suction velocity. The negative sign in the above equation indicates that the suction is towards the plate. The viscous dissipation in the temperature equation is assumed to be significant.

Denoting dimensional velocity components u^* , v^* and w^* in x^* , y^* and z^* axes respectively, the flow equations under the usual Boussinesq's approximation.

Continuity Equation

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0 \quad (2)$$

Momentum Equation

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} = g\beta(T^* - T_\infty^*) + \nu \left(\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}} \right) - \frac{\sigma B_0^2 u^*}{\rho} \quad (3)$$

$$\frac{\partial v^*}{\partial t^*} + v^* \frac{\partial v^*}{\partial y^*} + w^* \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial y^*} + \nu \left(\frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}} \right) \quad (4)$$

$$\frac{\partial w^*}{\partial t^*} + v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial z^*} + \nu \left(\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}} \right) - \frac{\sigma B_0^2 w^*}{\rho} \quad (5)$$

Energy Equation

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*} = \frac{k}{\rho C_p} \left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}} \right) - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} + \frac{Q^*}{\rho C_p} (T^* - T_\infty^*) + \frac{\mu}{\rho C_p} \Phi^* \quad (6)$$

where Φ^* is the viscous dissipation function given by

$$\Phi^* = 2 \left[\left(\frac{\partial v^*}{\partial y^*} \right)^2 + \left(\frac{\partial w^*}{\partial z^*} \right)^2 \right] + \left(\frac{\partial u^*}{\partial y^*} \right)^2 + \left(\frac{\partial w^*}{\partial y^*} + \frac{\partial v^*}{\partial z^*} \right)^2 + \left(\frac{\partial u^*}{\partial z^*} \right)^2$$

where ν is kinematic viscosity, g is gravity, P^* is fluid pressure, ρ is the density, κ -Thermal conductivity, T_w^* and T_∞^* are the wall and temperature outside the boundary layer, C_p is the specific heat and σ is electrical conductivity.

The similar boundary conditions are given as

$$\begin{aligned} u^* &= L^* \frac{\partial u^*}{\partial y^*}; & v^* &= -V_0 \left[1 + \epsilon \cos \left(\frac{\pi u_\infty z^*}{v} - ct^* \right) \right]; \\ w^* &= 0; & T^* &= T_w^* & \text{at } y^* &= 0 \\ u^* &= 0; & v^* &= -V_0; & w^* &= 0; & T^* &= T_\infty^*; \\ P^* &= P_\infty^* & \text{as } y^* &\rightarrow \infty \end{aligned} \quad (7)$$

where $L^* = \left(\frac{2-m}{m} \right) L$ and $L = \mu \left(\frac{\pi}{2P\rho} \right)^{1/2}$ is the mean free path, m is the Maxwell's reflection coefficient.

The optically thin limit for a non-gray gas near equilibrium and radioactive heat flux is represented by the following form:

$$\frac{\partial q_r^*}{\partial y^*} = 4(T^* - T_\infty^*) I^* \quad (8)$$

where $I^* = \int K_{\lambda w} \frac{\partial e_{b\lambda}}{\partial T^*} d\lambda$ is the absorption coefficient at

the plate, $e_{b\lambda}$ is plank Constant.

By introducing the non-dimensional scheme

$$\begin{aligned} y &= \frac{u_\infty y^*}{v}; & y &= \frac{u_\infty y^*}{v}; & t &= ct^*; & p &= \frac{P^*}{\rho U_\infty^2}; \\ u &= \frac{u^*}{u_\infty}; & v &= \frac{v^*}{u_\infty}; & w &= \frac{w^*}{u_\infty}; & \theta &= \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*} \end{aligned} \quad (10)$$

$\lambda = \frac{c\nu}{u_\infty^2}$, the frequency Parameter;

$Pr = \frac{\nu}{\alpha}$, the Prandtl Number;

$S = \frac{V_0}{u_\infty}$, the Suction Parameter;

$Gr = \frac{g\beta\nu(T_w^* - T_\infty^*)}{u_\infty^3}$; the Grashof Number;

$M = \frac{\sigma B_0^2 \nu}{\rho u_\infty^2}$, Hartmann Number;

$F = \frac{4\nu l}{\rho C_p u_\infty^2}$, Radiation Parameter;



$$Q = \frac{Q^* v}{\rho C_p u_\infty^2}, \text{ Heat Source;}$$

$$Ec = \frac{u_\infty^2}{C_p(T_w - T_\infty)}, \text{ the Eckert number;}$$

$$h = \frac{L^* u_\infty}{\nu}, \text{ Slip parameter due to main flow velocity}$$

The governing equations (2) - (6) can be written in non-dimensional form as

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{11}$$

$$\lambda \frac{\partial u}{\partial t} + \left(v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = Gr\theta + \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) - Mu \tag{12}$$

$$\lambda \frac{\partial v}{\partial t} + \left(v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial y} + \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \tag{13}$$

$$\lambda \frac{\partial w}{\partial t} + \left(v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - Mw \tag{14}$$

$$\lambda Pr \frac{\partial \theta}{\partial t} + Pr \left(v \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} \right) = \left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) - Pr(F - Q)\theta + Pr Ec\Phi \tag{15}$$

where Pr is the Prandtl Number and λ is the frequency parameter.

The corresponding boundary conditions in dimensionless form are

$$u = h \frac{\partial u}{\partial y}; v = -S[1 + \epsilon \cos(\pi z - t)]; w = 0; \theta = 1$$

at $y = 0$

$$u = 0; v = -S; w = 0; \theta = 0; \text{ as } y \rightarrow \infty \tag{16}$$

III. SOLUTION OF THE PROBLEM

When the amplitude of oscillation in the suction velocity is small ($\epsilon \ll 1$), we assume u, v, w, p and θ in the following form to solve the differential equations (11) - (16).

$$\begin{aligned} \text{Let } u(y, z, t) &= u_0(y) + \epsilon u_1(y, z, t) + \epsilon^2 u_2(y, z, t) + \dots \\ v(y, z, t) &= v_0(y) + \epsilon v_1(y, z, t) + \epsilon^2 v_2(y, z, t) + \dots \\ w(y, z, t) &= w_0(y) + \epsilon w_1(y, z, t) + \epsilon^2 w_2(y, z, t) + \dots \\ p(y, z, t) &= p_0(y) + \epsilon p_1(y, z, t) + \epsilon^2 p_2(y, z, t) + \dots \\ \theta(y, z, t) &= \theta_0(y) + \epsilon \theta_1(y, z, t) + \epsilon^2 \theta_2(y, z, t) + \dots \end{aligned} \tag{17}$$

When $\epsilon = 0$, the differential equations (11) - (15) pertaining to two dimensional flow are obtained as

$$v'_0 = 0 \tag{18}$$

$$u''_0 + Su'_0 - Mu_0 = -Gr\theta_0 \tag{19}$$

$$w''_0 + Su'_0 - Mw_0 = 0 \tag{20}$$

$$\theta''_0 + Pr S\theta'_0 - Pr(F - Q)\theta_0 = Pr Ec u_0'^2 \tag{21}$$

The boundary conditions are

$$u_0 = h \frac{\partial u_0}{\partial y}; v_0 = -S; w_0 = 0;$$

$$\theta_0 = 1 \text{ at } y = 0$$

$$u_0 = 0; v_0 = -S; w_0 = 0;$$

$$\theta_0 = 0 \text{ as } y \rightarrow \infty \tag{22}$$

The unsteady state equations are

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \tag{23}$$

$$\lambda \frac{\partial u_1}{\partial t} + \left(-S \frac{\partial u_1}{\partial y} + v_1 \frac{\partial u_0}{\partial y} \right) = Gr\theta_1 + \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} - Mu_1 \tag{24}$$

$$\lambda \frac{\partial v_1}{\partial t} - S \frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2} \tag{25}$$

$$\lambda \frac{\partial w_1}{\partial t} - S \frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2} - Mw_1 \tag{26}$$

$$\lambda Pr \frac{\partial \theta_1}{\partial t} + Pr \left(-S \frac{\partial \theta_1}{\partial y} + v_1 \frac{\partial \theta_0}{\partial y} \right) = \frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2} - (F - Q)Pr\theta_1 + 2Pr Ec u_0' u_1' \tag{27}$$

The corresponding boundary conditions become

$$u = h \frac{\partial u_1}{\partial y}; v_1 = -S[\cos(\pi z - t)]; w_1 = 0;$$

$$\theta_1 = 0 \text{ at } y = 0$$

$$u_1 = 0; v_1 = 0; w_1 = 0;$$

$$\theta_1 = 0; \text{ as } y \rightarrow \infty \tag{28}$$

Here, u_1, v_1, w_1, p_1 and θ_1 are assumed with complex notations as below whose real part will give the physical significance.

$$\begin{aligned} u_1(y, z, t) &= u_{11}(y)e^{i(\pi z - t)} \\ v_1(y, z, t) &= v_{11}(y)e^{i(\pi z - t)} \\ w_1(y, z, t) &= \frac{i}{\pi} v'_{11}(y)e^{i(\pi z - t)} \\ p_1(y, z, t) &= p_{11}(y)e^{i(\pi z - t)} \\ \theta_1(y, z, t) &= \theta_{11}(y)e^{i(\pi z - t)} \end{aligned} \tag{29}$$

By using (29) in equations (23) - (27) we get

$$v''_{11} + Sv'_{11} - (\pi^2 - i\lambda)v_{11} = p'_{11} \tag{30}$$

$$v'''_{11} + Sv''_{11} - (\pi^2 - i\lambda + M)v'_{11} = \pi^2 p_{11} \tag{31}$$

$$u''_{11} + Su'_{11} - (\pi^2 - i\lambda + M)u_{11} = v_{11}u'_0 - Gr\theta_{11} \tag{32}$$

$$\theta''_{11} + Pr S\theta'_{11} - (\pi^2 - i\lambda Pr + (F - Q)Pr)\theta_{11}$$



$$= \text{Pr}v_{11}\theta'_0 - 2\text{Pr}Ecu'_0u'_{11} \quad (33)$$

Boundary conditions are

$$\begin{aligned} u_{11} &= h \frac{\partial u_{11}}{\partial y}; & v_{11} &= -S; & \frac{\partial v_{11}}{\partial y} &= 0; \\ \theta_{11} &= 0 & \text{at } y &= 0 \\ u_{11} &= 0; & v_{11} &= 0; & \frac{\partial v_{11}}{\partial y} &= 0; \\ \theta_{11} &= 0 & \text{as } y &\rightarrow \infty \end{aligned} \quad (34)$$

In order to solve the coupled equations (19), (21), (32) and (33) under the corresponding boundary conditions (34), we further assume $Ec \ll 1$ for all incompressible fluids and it is assumed that the solution of the equations will be of the form:

$$u_0(y) = u_{00}(y) + Ecu_{01}(y) + o(Ec^2) \quad (35)$$

$$\theta_0(y) = \theta_{00}(y) + Ec\theta_{01}(y) + o(Ec^2) \quad (36)$$

$$u_{11}(y) = u_{110}(y) + Ecu_{111}(y) + o(Ec^2) \quad (37)$$

$$\theta_{11}(y) = \theta_{110}(y) + Ec\theta_{111}(y) + o(Ec^2) \quad (38)$$

Substituting (19) - (21) in the equation (35) - (36), (32)-(33) in the equations (37) and (38) respectively and neglecting the term of $o(Ec^2)$, the following differential equations with corresponding boundary conditions are derived.

$$u''_{00} + Su'_{00} - Mu_{00} = -Gr\theta_{00} \quad (39)$$

$$u''_{01} + Su'_{01} = -Gr\theta_{01} \quad (40)$$

$$\theta''_{00} + S\text{Pr}\theta'_{00} - \text{Pr}(F - Q)\theta_{00} = 0 \quad (42)$$

$$\theta''_{01} + S\text{Pr}\theta'_{01} - \text{Pr}(F - Q)\theta_{01} = -\text{Pr}(u'_{00})^2 \quad (43)$$

$$u''_{110} + Su'_{110} - (\pi^2 - i\lambda + M)u_{110} = v_{11}u'_{00} - Gr\theta_{110} \quad (44)$$

$$u''_{111} + Su'_{111} - (\pi^2 - i\lambda + M)u_{111} = v_{11}u'_{01} - Gr\theta_{111} \quad (45)$$

$$\theta''_{110} + \text{Pr}S\theta'_{110} - (\pi^2 - i\lambda\text{Pr} + (F - Q)\text{Pr})\theta_{110} = \text{Pr}v_{11}\theta'_{00} \quad (46)$$

$$\theta''_{111} + \text{Pr}S\theta'_{111} - (\pi^2 - i\lambda\text{Pr} + (F - Q)\text{Pr})\theta_{111} = \text{Pr}v_{11}\theta'_{01} - 2\text{Pr}u'_{00}u'_{110} \quad (47)$$

$$u_{00} = h \frac{\partial u_{00}}{\partial y}; \quad u_{01} = h \frac{\partial u_{01}}{\partial y}; \quad \theta_{00} = 1; \quad \theta_{01} = 0;$$

$$u_{110} = h \frac{\partial u_{110}}{\partial y}; \quad u_{111} = h \frac{\partial u_{111}}{\partial y}; \quad \theta_{110} = 0; \quad \theta_{111} = 0$$

at $y = 0$

$$u_{00} = 0; \quad u_{01} = 0; \quad \theta_{00} = 0; \quad \theta_{01} = 0$$

$$u_{110} = 0; \quad u_{111} = 0; \quad \theta_{110} = 0; \quad \theta_{111} = 0$$

$$\text{as } y \rightarrow \infty \quad (48)$$

The solutions of the equations (39) - (47) with the boundary conditions (48) are

$$\begin{aligned} u = & A_3e^{-r_3y} + A_5e^{-r_1y} + Ec(A_{11}e^{-r_7y} + A_{13}e^{-r_5y} + A_{14}e^{-2r_3y} \\ & + A_{15}e^{-2r_1y} + A_{16}e^{-(r_1+r_3)y}) + \epsilon(A_{24}e^{-r_{15}y} + A_{26}e^{-r_{13}y} \\ & + A_{27}e^{-(r_1+r_9)y} + A_{28}e^{-(r_1+r_{11})y} + A_{29}e^{-(r_3+r_9)y} \\ & + A_{30}e^{-(r_3+r_{11})y} + Ec(A_{45}e^{-r_{19}y} + A_{47}e^{-r_{17}y} \\ & + A_{48}e^{-(r_3+r_9)y} + A_{49}e^{-(r_7+r_9)y} + A_{50}e^{-(r_3+r_{11})y} \\ & + A_{51}e^{-(r_7+r_{11})y} + A_{52}e^{-(r_3+r_{15})y} + A_{53}e^{-(r_3+r_{13})y} \\ & + A_{54}e^{-(r_1+r_{15})y} + A_{55}e^{-(r_1+r_{13})y} + A_{56}e^{-(2r_3+r_9)y} \\ & + A_{57}e^{-(2r_3+r_{11})y} + A_{58}e^{-(2r_1+r_9)y} + A_{59}e^{-(2r_1+r_{11})y} \\ & + A_{60}e^{-(r_1+r_3+r_9)y} + A_{61}e^{-(r_1+r_3+r_{11})y})e^{i(\pi z - t)} + O(\epsilon^2) \end{aligned} \quad (49)$$

$$v = -S + \epsilon(A_{17}e^{-r_9y} + A_{19}e^{-r_{11}y})e^{i(\pi z - t)} + O(\epsilon^2) \quad (50)$$

$$w = -\frac{i}{\pi}(r_9A_{17}e^{-r_9y} + r_{11}A_{19}e^{-r_{11}y})e^{i(\pi z - t)} + O(\epsilon^2) \quad (51)$$

$$\begin{aligned} \theta = & e^{-r_1y} + Ec(A_6e^{-r_5y} + A_8e^{-2r_3y} + A_9e^{-2r_1y} \\ & + A_{10}e^{-(r_1+r_3)y}) + \epsilon(A_{20}e^{-r_{13}y} + A_{22}e^{-(r_1+r_9)y} \\ & + A_{23}e^{-(r_1+r_{11})y} + Ec(A_{31}e^{-r_{17}y} + A_{33}e^{-(r_3+r_9)y} \\ & + A_{34}e^{-(2r_3+r_9)y} + A_{35}e^{-(r_9+2r_1)y} + A_{36}e^{-(2r_3+r_{11})y} \\ & + A_{37}e^{-(r_1+2r_1)y} + A_{38}e^{-(r_5+r_{11})y} + A_{39}e^{-(r_3+r_{15})y} \\ & + A_{40}e^{-(r_3+r_{13})y} + A_{41}e^{-(r_1+r_{15})y} + A_{42}e^{-(r_1+r_{13})y} \\ & + A_{43}e^{-(r_1+r_3+r_9)y} + A_{44}e^{-(r_1+r_3+r_{11})y})e^{i(\pi z - t)} \\ & + O(\epsilon^2) \end{aligned} \quad (52)$$

where r_5, r_6, r_7 and r_8 are the roots of the equation

$$r^4 + Sr^3 - (2\pi^2 - i\lambda + M)r^2 - S\pi^2r + (\pi^2 - i\lambda)\pi^2 = 0$$

and the constants are given in the Appendix

SKIN FRICTION AND HEAT FLUX

The skin friction at the wall is given as

$$\begin{aligned} \tau &= \left(\frac{du}{dy}\right)_{y=0} = \left(\frac{du_0}{dy}\right)_{y=0} + \epsilon \left(\frac{du_1}{dy}\right)_{y=0} + O(\epsilon^2) \\ &= DE1 + \epsilon DE2 + O(\epsilon^2) \end{aligned}$$

The heat flux in terms of Nusselt Number is given

as

$$\begin{aligned} Nu &= \left(\frac{d\theta}{dy}\right)_{y=0} = \left(\frac{d\theta_0}{dy}\right)_{y=0} + \epsilon \left(\frac{d\theta_1}{dy}\right)_{y=0} + O(\epsilon^2) \\ &= DF1 + \epsilon DF2 + O(\epsilon^2) \end{aligned}$$

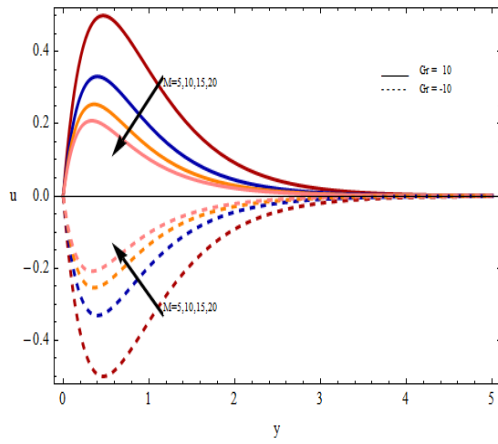


Figure 2: Main Velocity u versus y for $S = 1.0$, $\lambda = 10$, $z = 0.0$, $\epsilon = 0.05$, $t = 0.2$, $h = 0$, $Pr = 0.71$, $F = 2$, $Q = 0$, $Ec = 0$

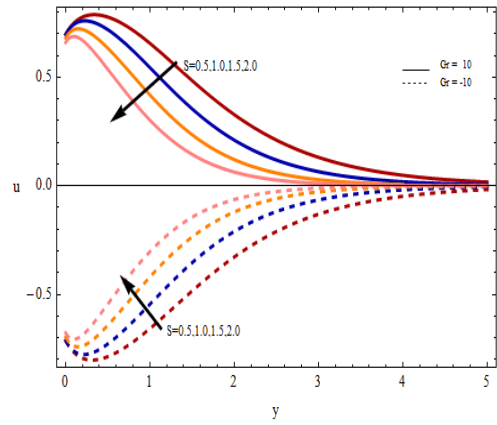


Figure 5 : Main Velocity u versus y for $F = 2$, $\lambda = 10$, $z = 0.0$, $\epsilon = 0.05$, $t = 0.2$, $h = 1$, $Pr = 0.71$, $M = 10$, $Q = 1$, $Ec = 10$

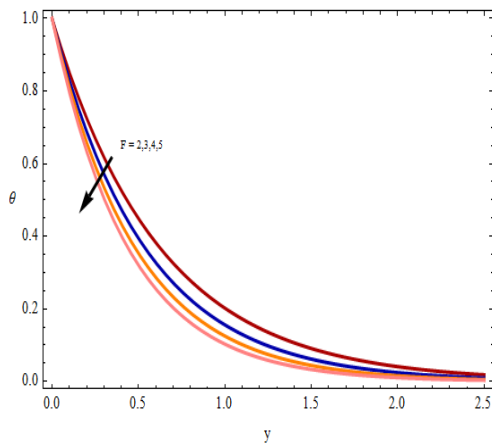


Figure 3: Temperature θ versus y for $S = 1.0$, $\lambda = 10$, $z = 0.0$, $\epsilon = 0.05$, $t = 0.2$, $h = 0$, $Pr = 0.71$, $M = 10$, $Q = 0$, $Ec = 0$

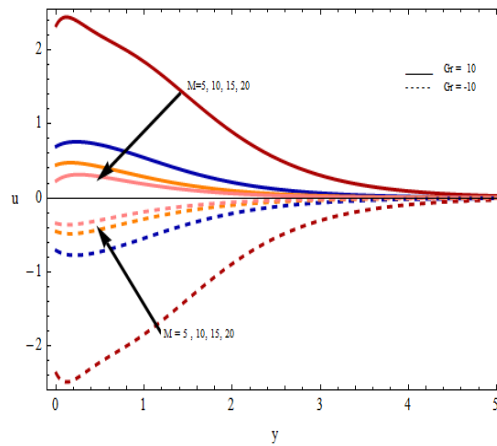


Figure 6 : Main Velocity u versus y for $S = 1.0$, $\lambda = 10$, $z = 0.0$, $\epsilon = 0.05$, $t = 0.2$, $h = 1$, $Pr = 0.71$, $F = 2$, $Q = 1$, $Ec = 10$

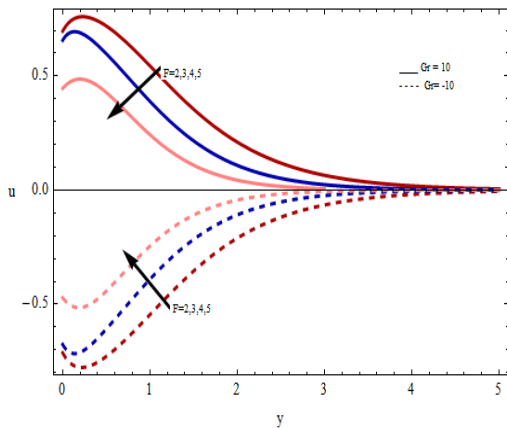


Figure 4 : Main Velocity u versus y for $S = 1.0$, $\lambda = 10$, $z = 0.0$, $\epsilon = 0.05$, $t = 0.2$, $h = 1$, $Pr = 0.71$, $M = 10$, $Q = 1$, $Ec = 10$

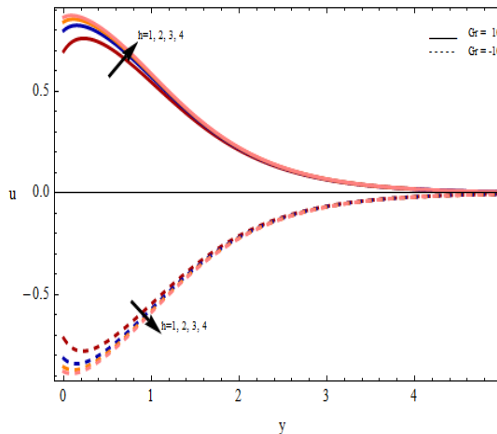


Figure 7 : Main Velocity u versus y for $S = 1.0$, $\lambda = 10$, $z = 0.0$, $\epsilon = 0.05$, $t = 0.2$, $M = 10$, $Pr = 0.71$, $F = 2$, $Q = 1$, $Ec = 10$

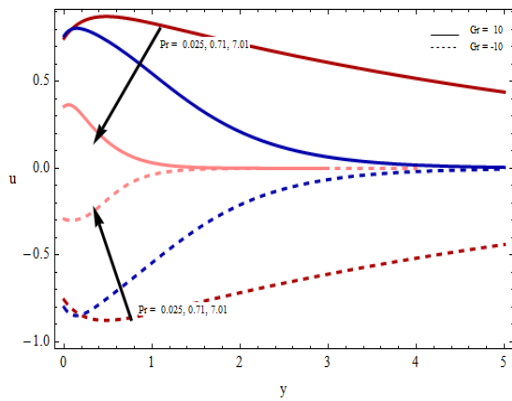


Figure 8 : Main Velocity u versus y for $S = 1.0$, $\lambda = 10$, $z = 0.0$, $\epsilon = 0.05$, $t = 0.2$, $h = 1$, $M = 10$, $F = 2$, $Q = 1$, $Ec = 10$

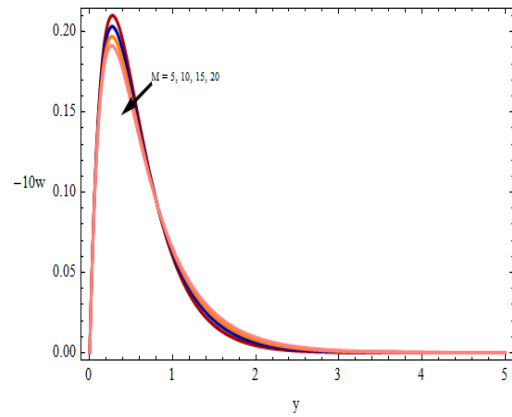


Figure 11 : Cross Velocity $-10w$ versus y for $S = 1$, $\lambda = 10$, $z = 0.0$, $\epsilon = 0.05$, $t = 0.2$, $Pr = 0.71$, $F = 2$, $Q = 1$, $Ec = 10$

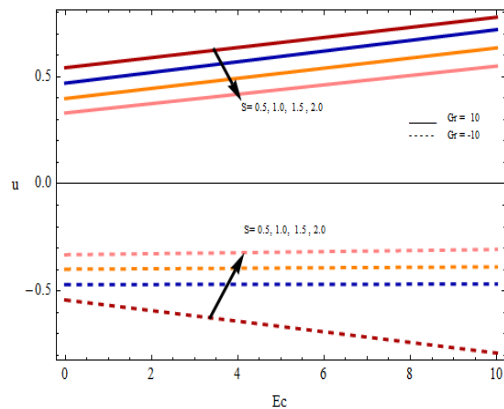


Figure 9 : Main Velocity u versus y for $Pr = 0.71$, $\lambda = 10$, $z = 0.0$, $\epsilon = 0.05$, $t = 0.2$, $h = 1$, $M = 10$, $F = 2$, $Q = 1$ at $y = 0.5$

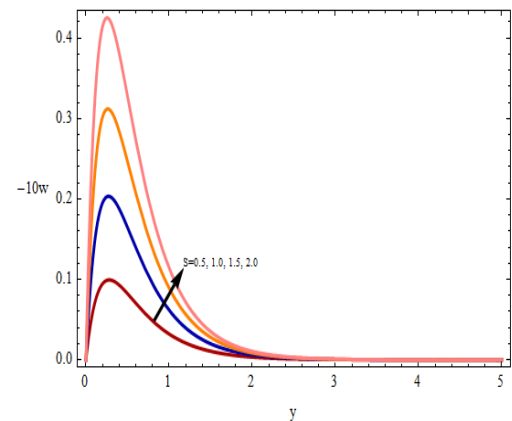


Figure 12 : Cross Velocity $-10w$ versus y for $M = 10$, $\lambda = 10$, $z = 0.0$, $\epsilon = 0.05$, $t = 0.2$, $Pr = 0.71$, $F = 2$, $Q = 1$, $Ec = 10$

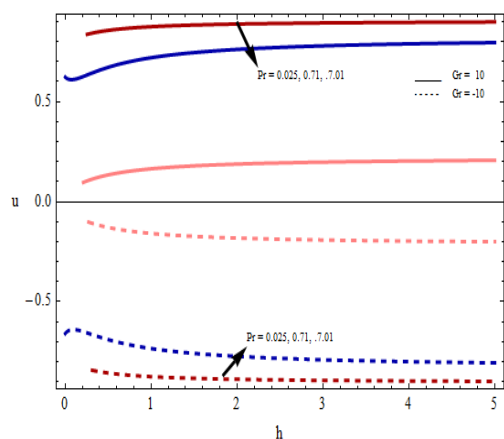


Figure 10 : Main Velocity u versus y for $S = 1$, $\lambda = 10$, $z = 0.0$, $\epsilon = 0.05$, $t = 0.2$, $M = 10$, $F = 2$, $Q = 1$, $Ec = 10$ at $y = 0.5$

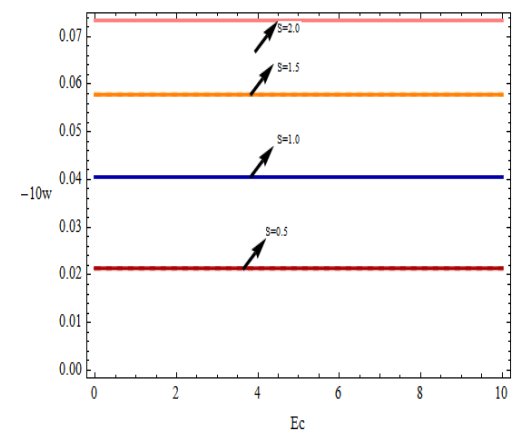


Figure 13 : Cross Velocity $-10w$ versus Ec for $M = 10$, $\lambda = 10$, $z = 0.0$, $\epsilon = 0.05$, $t = 0.2$, $Pr = 0.71$, $F = 2$, $Q = 1$ at $y = 0.5$

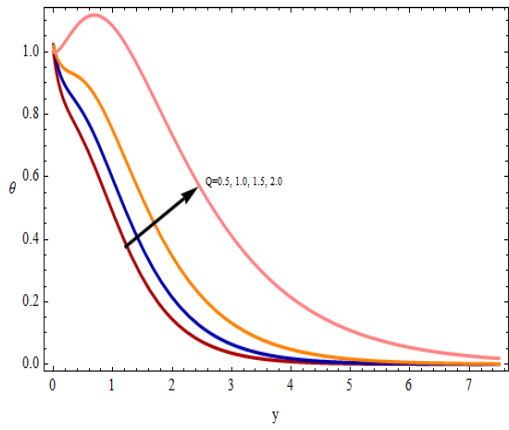


Figure 14: Temperature θ versus y for $F = 2, \lambda = 10, z = 0.0$

$\epsilon = 0.05, t = 0.2, h = 1, Pr = 0.71, M = 10, S = 1, Gr = 10$
 $Ec = 10$

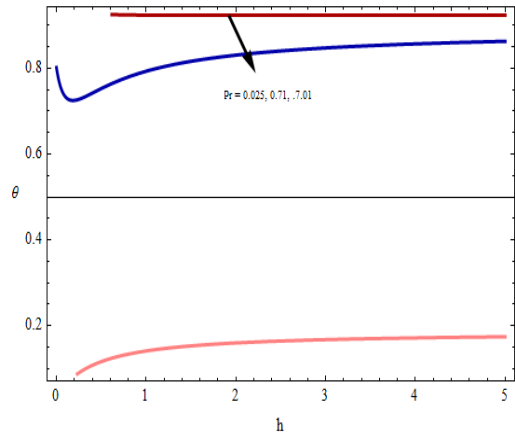


Figure 17: Temperature θ versus h for $F = 2, \lambda = 10, z = 0.0$

$\epsilon = 0.05, t = 0.2, S = 1, M = 10, Q = 1, Ec = 10,$
 $Gr = 10$ at $y = 0.5$

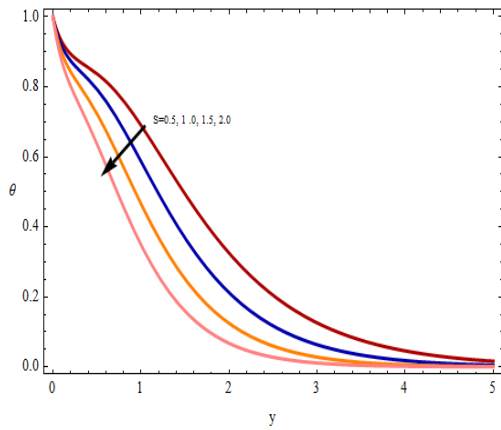


Figure 15: Temperature θ versus y for $F = 2, \lambda = 10, z = 0.0$

$\epsilon = 0.05, t = 0.2, h = 1, Pr = 0.71, M = 10, Q = 1,$
 $Gr = 10, Ec = 10$

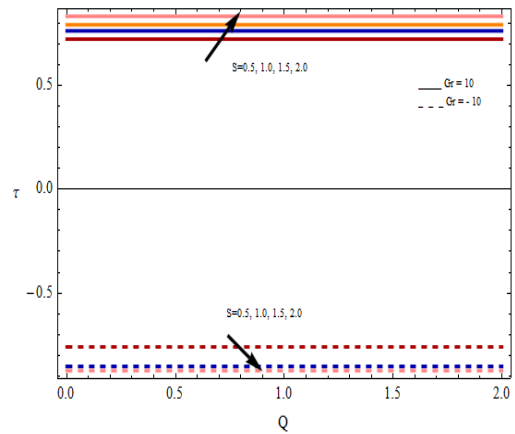


Figure 18: Skin friction τ versus Q for $F = 2, \lambda = 10,$

$z = 0.2$
 $\epsilon = 0.2, t = 0.2, h = 1, Pr = 0.71, M = 10, Ec = 10$ at
 $y = 0$

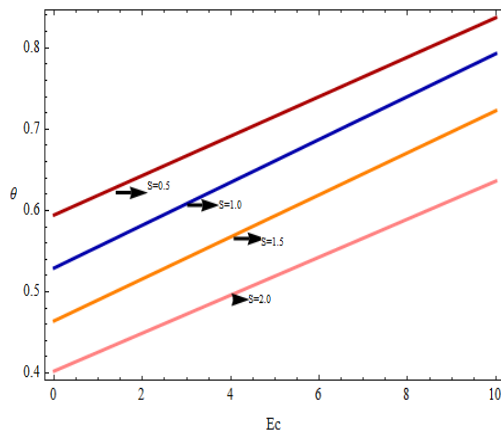


Figure 16: Temperature θ versus Ec for $F = 2, \lambda = 10,$

$z = 0.0, \epsilon = 0.05, t = 0.2, h = 1, Pr = 0.71, M = 10,$
 $Q = 1, Gr = 10$ at $y = 0.5$

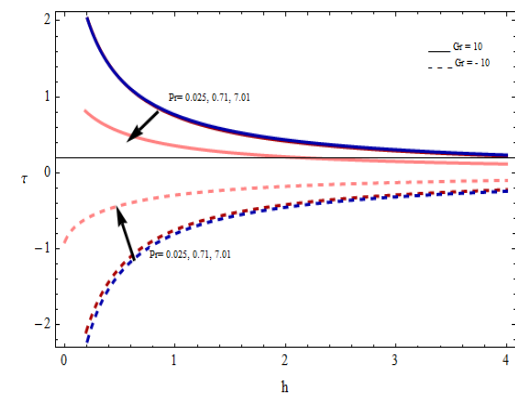


Figure 19: Skin friction τ versus h for $F = 2, \lambda = 10, z = 0.2$

$\epsilon = 0.2, t = 0.2, S = 1, M = 10, Q = 1, Ec = 10$ at $y = 0$

IV. NUMERICAL RESULTS

In order to obtain the physical insight of the problem, the main flow, cross flow and skin friction are studied as a function of various non dimensional parameters such as Grashof Number Gr , Prandtl Number Pr , Suction parameter S , Frequency parameter λ , Hartmann number M , Eckert number Ec , Heat Source Q , Radiation parameter F and slip parameter due to main flow velocity h . The effect of various flow parameters on velocity and temperature profile, Skin friction and Nusselt number are calculated, these flow properties are figuratively explained as functions of various parameters.

To verify the validity of the expressions derived in the previous part the main flow velocity u and temperature in Figures (2) and (3) for vanishing slip parameter, the dimensionless parameter Gr is taken as positive and negative. The positive value corresponds to cooling plate by free convection currents and the negative value corresponds to the hot plate. It can be seen from the Figure (2) that the main flow velocity is analyzed for both cooling and heating plate. Increasing Hartmann number M decreases the main flow velocity for cooling plate and increases for heating of the plate. Figure (3) shows that increasing Radiation parameter F decreases the temperature profile. These results have a qualitative and quantitative agreement with the results achieved by Guria and Jana [5].

In Figures (4) - (10), the main flow velocity is given as a function of various parameters such as Ec and h . Increase in radiation Parameter F , Hartmann number M , Suction parameter S and Prandtl Number Pr decreases the velocity profile for cooling plate and increase in heating plate. From Figure (7) increasing h increases the main flow velocity.

Figures (11) - (13) presents the behaviour of the cross flow velocity as a function of various non dimensional numbers. Increasing Suction parameter S and Eckert Number Ec increases the cross flow velocity whereas decreases the cross flow velocity by increasing Hartmann number M .

Figures (14) - (17) depict the behaviour of the temperature profile which is plotted as a function of y and a function of various parameters such as h and Ec . Increasing Eckert number Ec , suction parameter, slip parameter h are found to decrease the temperature. Increase in heat source parameter Q increases the temperature.

Another significant flow characteristic, skin friction at the porous plate is shown in Figures (18) - (20). The values of τ are shown for both positive and negative Gr . For a cooled plate, Prandtl Number Pr decrease the wall shear stress, while for a hot plate increase in these parameters increase the skin friction. Increase in the Suction parameter S increase the skin friction.

From Figures (21) - (22), the rate of heat transfer is found to increase with increasing suction parameter S and Prandtl number Pr at $y = 0$.

V. CONCLUSION

The work of Guria and Jana [5] have been extended to analyze the effect of heat

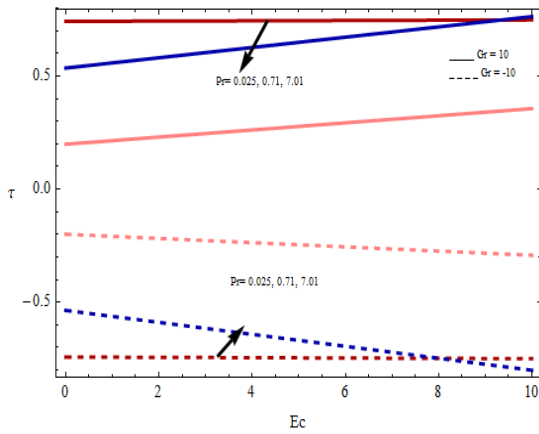


Figure 20: Skin friction τ versus h for $F = 2, \lambda = 10, z = 0.2, \epsilon = 0.2, t = 0.2, S = 1, M = 10, Q = 1, h = 1$ at $y=0$

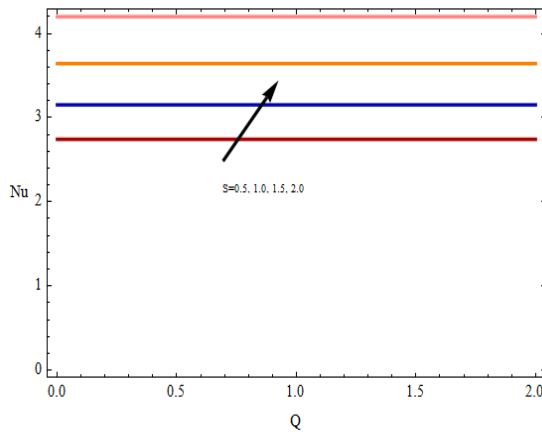


Figure 21: Skin friction τ versus h for $F = 2, \lambda = 10, z = 0.2, \epsilon = 0.2, t = 0.2, Pr = 0.71, M = 10, h = 1, Ec = 10, Gr = 10$ at $y=0$

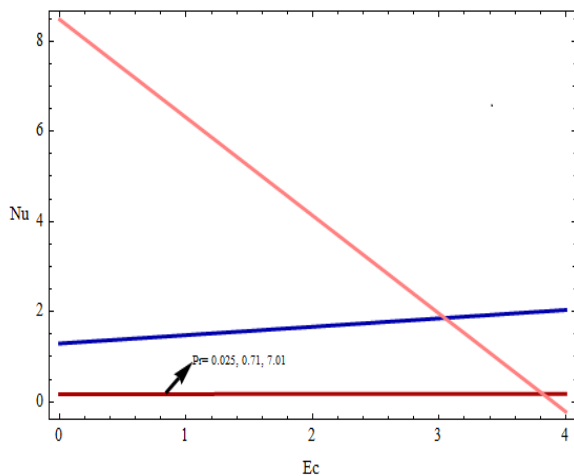


Figure 22: Skin friction τ versus h for $F = 2, \lambda = 10, z = 0.2, \epsilon = 0.2, t = 0.2, S = 1, M = 10, Q = 1, h = 1, Gr = 10$ at $y=0$

source, viscous dissipation and slip flow regime for significant flow velocity and temperature on unsteady three dimensional MHD flow of a viscous fluid precede a vertical porous plate with periodic suction at the stationary plate. The periodic suction velocity is taken to be time dependent and perpendicular to the flow direction. Analytical expressions were used to calculate temperature, main flow and cross flow velocity profile. The skin friction coefficient and heat transfer coefficient are also found. Therefore it is concluded that

- Increase in radiation Parameter F , Hartmann number M , Suction parameter S and Prandtl Number Pr decreases the velocity profile for cooling plate and increase in heating plate.
- Increase in Suction parameter S and Eckert Number Ec also increases the cross flow velocity whereas decreases the cross flow velocity by increasing Hartmann number M .
- Increasing Eckert number Ec , suction parameter, slip parameter h is seen to decrease the temperature. Increase in heat source parameter Q increases the temperature.
- Increase in the Suction parameter S increase the skin friction.
- The rate of heat transfer is found to increase with increasing suction parameter S and Prandtl number Pr at $y = 0$.

Appendix

$$r_1, r_5 = \frac{S Pr + \sqrt{(S Pr)^2 + 4 Pr (F - Q)}}{2}; \quad r_3, r_7 = \frac{S + \sqrt{S^2 + 4 M}}{2}$$

$$r_{13}, r_{17} = \frac{S Pr + \sqrt{(S Pr)^2 + 4(\pi^2 - i\lambda Pr + F Pr - Q Pr)}}{2};$$

$$r_{15}, r_{19} = \frac{S + \sqrt{S^2 + 4(\pi^2 - i\lambda + M)}}{2}; \quad A_3 = \frac{-A_5(1 + hr_1)}{1 + hr_3};$$

$$A_5 = \frac{-Gr}{r_1^2 - Sr_1 - M}; \quad A_6 = -(A_8 + A_9 + A_{10})$$

$$A_8 = \frac{-Pr(A_3 r_3)^2}{(2r_3)^2 - 2r_3 S Pr - Pr(F - Q)}$$

$$A_{10} = \frac{-2Pr A_3 A_5 r_1 r_3}{(r_1 + r_3)^2 - S Pr(r_1 + r_3) - Pr(F - Q)}$$

$$A_{11} = \frac{-(1 + hr_5)A_{13} - (1 + 2hr_3)A_{14} - (1 + 2hr_1)A_{15} - (1 + h(r_1 + r_3))A_{16}}{(1 + hr_7)}$$

$$A_{13} = \frac{-GrA_6}{(r_5)^2 - Sr_5 - M}; \quad A_{14} = \frac{-GrA_8}{(2r_3)^2 - 2Sr_3 - M}$$

$$A_{15} = \frac{-GrA_9}{(2r_1)^2 - 2Sr_1 - M}; \quad A_{16} = \frac{-GrA_{10}}{(r_1 + r_3)^2 - S(r_1 + r_3) - M}$$

$$A_{17} = \frac{-Sr_{11}}{r_{11} - r_9}; \quad A_{19} = \frac{Sr_9}{r_{11} - r_9}; \quad A_{20} = -(A_{22} + A_{23});$$

$$A_{22} = \frac{-Pr A_1 A_{17} r_1}{(r_1 + r_9)^2 - S Pr(r_1 + r_9) - (\pi^2 - i\lambda Pr + F Pr - Q Pr)}$$

$$A_{23} = \frac{-Pr A_1 A_{19} r_1}{(r_1 + r_{11})^2 - S Pr(r_1 + r_{11}) - (\pi^2 - i\lambda Pr + F Pr - Q Pr)}$$

$$A_{24} = \frac{1}{(1 + hr_{15})} \begin{pmatrix} -(1 + hr_{13})A_{26} - (1 + h(r_1 + r_9))A_{27} \\ -(1 + h(r_1 + r_{11}))A_{28} - (1 + h(r_3 + r_9))A_{29} \\ -(1 + h(r_3 + r_{11}))A_{30} \end{pmatrix}$$

$$A_{26} = \frac{-GrA_{20}}{r_{13}^2 - Sr_{13} - (\pi^2 - i\lambda + M)}$$

$$A_{27} = \frac{-r_1 A_5 A_{17} - GrA_{22}}{(r_1 + r_9)^2 - S(r_1 + r_9) - (\pi^2 - i\lambda + M)}$$

$$A_{28} = \frac{-r_1 A_5 A_{19} - GrA_{23}}{(r_1 + r_{11})^2 - S(r_1 + r_{11}) - (\pi^2 - i\lambda + M)}$$

$$A_{29} = \frac{-r_3 A_3 A_{17}}{(r_3 + r_9)^2 - S(r_3 + r_9) - (\pi^2 - i\lambda + M)}$$

$$A_{30} = \frac{-r_3 A_3 A_{19}}{(r_3 + r_{11})^2 - S(r_3 + r_{11}) - (\pi^2 - i\lambda + M)}$$

$$A_{31} = \left(\begin{matrix} A_{33} + A_{34} + A_{35} + A_{36} + A_{37} + A_{38} \\ + A_{39} + A_{40} + A_{41} + A_{42} + A_{43} + A_{44} \end{matrix} \right);$$

$$A_{33} = \frac{-Pr A_6 A_{17} r_5}{(r_5 + r_9)^2 - S Pr(r_5 + r_9) - (\pi^2 - i\lambda Pr + F Pr - Q Pr)}$$

$$A_{34} = \frac{-2A_8 A_{17} r_3 Pr - 2Pr r_3 A_3 (r_3 + r_9)}{(2r_3 + r_9)^2 - S Pr(2r_3 + r_9) - (\pi^2 - i\lambda Pr + F Pr - Q Pr)}$$

$$A_{35} = \frac{-2A_9 A_{17} r_1 Pr - 2Pr r_1 A_5 A_{27} (r_1 + r_9)}{(2r_1 + r_9)^2 - S Pr(2r_1 + r_9) - (\pi^2 - i\lambda Pr + F Pr - Q Pr)}$$

$$A_{36} = \frac{-2A_8 A_{19} r_3 Pr - 2Pr r_3 A_3 A_{30} (r_3 + r_{11})}{(2r_3 + r_{11})^2 - S Pr(2r_3 + r_{11}) - (\pi^2 - i\lambda Pr + F Pr - Q Pr)}$$

$$A_{37} = \frac{-2A_9 A_{19} r_1 Pr - 2Pr r_1 A_5 A_{28} (r_1 + r_{11})}{(2r_1 + r_{11})^2 - S Pr(2r_1 + r_{11}) - (\pi^2 - i\lambda Pr + F Pr - Q Pr)}$$

$$A_{38} = \frac{-Pr A_6 A_{19} r_5}{(r_5 + r_{11})^2 - S Pr(r_5 + r_{11}) - (\pi^2 - i\lambda Pr + F Pr - Q Pr)}$$

$$A_{39} = \frac{-2Pr A_3 A_{24} r_3 r_{15}}{(r_3 + r_{15})^2 - S Pr(r_3 + r_{15}) - (\pi^2 - i\lambda Pr + F Pr - Q Pr)}$$

$$A_{40} = \frac{-2Pr A_3 A_{26} r_3 r_{13}}{(r_3 + r_{13})^2 - S Pr(r_3 + r_{13}) - (\pi^2 - i\lambda Pr + F Pr - Q Pr)}$$

$$A_{41} = \frac{-2Pr A_5 A_{24} r_1 r_{15}}{(r_1 + r_{15})^2 - S Pr(r_1 + r_{15}) - (\pi^2 - i\lambda Pr + F Pr - Q Pr)}$$

$$A_{42} = \frac{-2Pr A_5 A_{26} r_1 r_{13}}{(r_1 + r_{13})^2 - S Pr(r_1 + r_{13}) - (\pi^2 - i\lambda Pr + F Pr - Q Pr)}$$

$$A_{43} = \frac{-(r_1 + r_3) Pr A_{10} A_{17} - 2(r_1 + r_9) Pr r_3 A_3 A_{27} - 2(r_3 + r_9) Pr r_1 A_5 A_{29}}{(r_1 + r_3 + r_9)^2 - S Pr(r_1 + r_3 + r_9) - (\pi^2 - i\lambda Pr + F Pr - Q Pr)}$$

$$A_{44} = \frac{-(r_1 + r_3) Pr A_{10} A_{19} - 2(r_1 + r_{11}) Pr r_3 A_3 A_{28} - 2(r_3 + r_{11}) Pr r_1 A_5 A_{30}}{(r_1 + r_3 + r_{11})^2 - S Pr(r_1 + r_3 + r_{11}) - (\pi^2 - i\lambda Pr + F Pr - Q Pr)}$$

$$A_{47} = \frac{-GrA_{31}}{(r_{17})^2 - S(r_{17}) - (\pi^2 - i\lambda + M)}$$

$$A_{48} = \frac{-r_5 A_{13} A_{17} - GrA_{33}}{(r_5 + r_9)^2 - S(r_5 + r_9) - (\pi^2 - i\lambda + M)}$$

$$A_{49} = \frac{-r_7 A_{11} A_{17}}{(r_7 + r_9)^2 - S(r_7 + r_9) - (\pi^2 - i\lambda + M)}$$

$$A_{50} = \frac{-r_5 A_{13} A_{19} - GrA_{38}}{(r_5 + r_{11})^2 - S(r_5 + r_{11}) - (\pi^2 - i\lambda + M)}$$

$$A_{51} = \frac{-r_7 A_{11} A_{19}}{(r_7 + r_{11})^2 - S(r_7 + r_{11}) - (\pi^2 - i\lambda + M)}$$

$$A_{52} = \frac{-GrA_{39}}{(r_3 + r_{15})^2 - S(r_3 + r_{15}) - (\pi^2 - i\lambda + M)}$$



$$A_{53} = \frac{-GrA_{40}}{(r_3 + r_{13})^2 - S(r_3 + r_{13}) - (\pi^2 - i\lambda + M)};$$

$$A_{54} = \frac{-GrA_{41}}{(r_1 + r_{15})^2 - S(r_1 + r_{15}) - (\pi^2 - i\lambda + M)};$$

$$A_{55} = \frac{-GrA_{42}}{(r_1 + r_{13})^2 - S(r_1 + r_{13}) - (\pi^2 - i\lambda + M)};$$

$$A_{56} = \frac{-2r_3A_{14}A_{17} - GrA_{34}}{(2r_3 + r_9)^2 - S(2r_3 + r_9) - (\pi^2 - i\lambda + M)};$$

$$A_{57} = \frac{-2r_3A_{14}A_{19} - GrA_{36}}{(2r_3 + r_{11})^2 - S(2r_3 + r_{11}) - (\pi^2 - i\lambda + M)};$$

$$A_{58} = \frac{-2r_1A_{15}A_{17} - GrA_{35}}{(2r_1 + r_9)^2 - S(2r_1 + r_9) - (\pi^2 - i\lambda + M)};$$

$$A_{59} = \frac{-2r_3A_{15}A_{19} - GrA_{37}}{(2r_1 + r_{11})^2 - S(2r_1 + r_{11}) - (\pi^2 - i\lambda + M)};$$

$$A_{60} = \frac{-(r_1 + r_3)A_{16}A_{17} - GrA_{43}}{(r_1 + r_3 + r_9)^2 - S(r_1 + r_3 + r_9) - (\pi^2 - i\lambda + M)};$$

$$A_{61} = \frac{-(r_1 + r_3)A_{16}A_{19} - GrA_{44}}{(r_1 + r_3 + r_{11})^2 - S(r_1 + r_3 + r_{11}) - (\pi^2 - i\lambda + M)};$$

$$DE1 = \frac{-r_3A_3 - r_1A_5 + Ec(-r_7A_{11} - r_5A_{13} - 2r_3A_{14})}{-2r_1A_{15} - (r_1 + r_3)A_{16}};$$

$$DE2 = \frac{-r_1A_{24} - r_3A_{26} - (r_1 + r_9)A_{27} - (r_1 + r_{11})A_{28}}{- (r_3 + r_9)A_{29} - (r_3 + r_{11})A_{30} + Ec(-r_9A_{45} - r_1A_{47} - (r_5 + r_9)A_{48} - (r_7 + r_9)A_{49} - (r_5 + r_{11})A_{50} - (r_7 + r_{11})A_{51} - (r_3 + r_{15})A_{52} - (r_3 + r_{13})A_{53} - (r_1 + r_{15})A_{54} - (r_1 + r_{13})A_{55} - (2r_3 + r_9)A_{56} - (2r_3 + r_{11})A_{57} - (2r_1 + r_9)A_{58} - (2r_1 + r_{11})A_{59} - (r_1 + r_3 + r_9)A_{60} - (r_1 + r_3 + r_{11})A_{61})}$$

$$DF1 = \frac{-r_1 + Ec(-r_5A_6 - 2r_3A_8 - 2r_1A_9 - (r_1 + r_3)A_{10})}{-r_1A_{20} - (r_1 + r_9)A_{22} - (r_1 + r_{11})A_{23}}$$

$$DF2 = \frac{+ Ec(-r_7A_{31} - (r_5 + r_9)A_{33} - (2r_3 + r_9)A_{34} - (r_9 + 2r_1)A_{35} - (2r_3 + r_{11})A_{36} - (r_{11} + 2r_1)A_{37} - (r_5 + r_{11})A_{38} - (r_3 + r_{15})A_{39} - (r_3 + r_{13})A_{40} - (r_1 + r_{15})A_{41} - (r_1 + r_{13})A_{42} - (r_1 + r_3 + r_9)A_{43} - (r_1 + r_3 + r_{11})A_{44})}{}$$

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