

# Effects of Inclination of the Plate Embedded in Porous Media on Radiative and Chemically Reactive MHD Convection

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**Abstract:** This work is focused on the numerical study of thermodiffusion, inclination of the plate, order of chemical reaction, Diffusion-thermo and thermal radiation effects on a steady magnetohydrodynamic convective flow over an inclined plate in a porous medium under the influence of viscous dissipation along with the application of heat generation/absorption effects. The partial differential equations governing the fluid flow are transformed into coupled non dimensional ordinary differential equations with the help of similarity transformations. Suitable codes in MATLAB's built in solver *bvp4c*, which is a highly accurate and efficient solver of MATLAB, are developed to solve these coupled ordinary differential equations numerically. The behaviour of the fluid velocity, temperature and species concentration for variations in the various thermo-physical parameters are illustrated via graphs. From the numerical results it is evident that the heat and mass transfer of the fluid are significantly influenced by the order of chemical reaction, thermal radiation, inclination of the plate, Soret and Dufour effects. Results obtained in this paper may be useful in the field of chemical industries, chemical engineering, petroleum engineering. Gas separating instruments can be installed in big cities as an engineering application so that harmful pollutants can be removed which are present in small quantities mixed with air.

**Keywords:** Mass transfer; porous medium; inclined plate; radiation; chemical reaction.

## NOMENCLATURE

$u$  – Velocity component of the fluid along  $x$  axis  
 $v$  – Velocity component of the fluid along  $y$  axis  
 $C_p$  – Specific heat at constant pressure  
 $g$  – Acceleration due to gravity  
 $K_T$  – Thermal diffusion ratio  
 $K_1$  – Dimensional chemical reaction parameter  
 $T_m$  – Mean fluid temperature  
 $C_S$  – Concentration susceptibility  
 $C$  – Concentration  
 $C_\infty$  – Concentration at static condition  
 $T$  – Temperature

$T_\infty$  – Temperature at static condition  
 $n$  – Order of chemical reaction  
 $B_0$  – Magnetic field strength  
 $Q_0$  – Heat generation / absorption coefficient  
 $U_\infty$  – Free stream velocity  
 $q_r$  – Radiative heat flux  
 $D_m$  – Mass diffusivity  
 $M$  – Hartmann number i.e. magnetic parameter  
 $D_f$  – Dufour number

$Sr$  – Soret number  
 $Pr$  – Prandtl number  
 $G_T$  – Thermal Grashof number  
 $G_C$  – Solutal Grashof number  
 $Da$  – Permeability parameter  
 $Ec$  – Eckert number  
 $Sc$  – Schmidt number  
 $N$  – viscous dissipation parameter

## Greek symbols

$\theta$  – Dimensionless temperature  
 $\phi$  – Dimensionless concentration  
 $\rho$  – Density of the fluid  
 $\nu$  – Kinematic viscosity  
 $\beta_T$  – Coefficient of thermal expansion  
 $\beta_C$  – Coefficient of concentration expansion  
 $\alpha$  – Thermal diffusivity  
 $k^*$  – Coefficient of mean absorption  
 $\mu$  – Fluid dynamic viscosity  
 $\sigma$  – Electrical conductivity of the fluid  
 $\gamma$  – Dimensionless chemical reaction parameter  
 $\omega$  – Acute angle made by the plate with the vertical  
 $\delta$  – Dimensionless heat source parameter  
 $\lambda$  – Thermal conductivity  
 $\kappa$  – Permeability of the porous medium  
 $\sigma^*$  – Stefan-Boltzman constant .

## I. INTRODUCTION

The magnetohydrodynamic fluid flow through a porous medium has a large variety of applications in engineering and industrial fields such as nuclear reactors, MHD pumps, MHD bearings, enhanced oil recovery, underground energy transport, etc. Ingham et. al. **Error! Reference source not found.** and Nield et. al. **Error! Reference source not found.** in their series of books have made extensive analysis on the fundamentals and experimental research in porous media.

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The Soret effect in porous media plays a significant role in the field of chemical engineering, petroleum engineering, nuclear waste disposal research, medicines and ceramics. It is also known as thermodiffusion which have been discovered by Charles Soret. Diffusion-thermo effect also known as Dufour effect is the energy flux created due to a composition gradient. Postelnicu had discussed the natural convective flow by taking the Dufour effect and Soret effect. Sharma et. al. have investigated the effects of Soret number and Dufour number on separating the species of a binary fluid mixture over a plate under a porous media. The chemically reactive mass transfer are of relevant importance in various industrial processes for instance polymer production, damage of crops caused by freezing, the manufacturing of ceramics or glassware, etc. In fluid flow problems the chemical reactions are characterized mainly into two types viz. homogeneous or heterogeneous. The reaction is heterogeneous if the reaction occurs at an interface and the chemical reaction is homogeneous reaction if it occurs within the solution. In various chemical reactions, the order of reaction depends upon the species concentration. Sharma and Borgohain have elaborately studied the chemical reaction rate and Soret number effects on magnetohydrodynamic flow over various geometries embedded in porous medium. The radiation effects with natural convection steady flow along a plate were analyzed by researchers such as Cess. Singh. has analyzed the effects of viscous dissipation and porosity on heat transfer and mass transfer past an inclined plate. More recently, Subhakanthi and Reddy have analyzed the mass transfer problems in chemically reactive magnetohydrodynamic flow over inclined plate in porous media in addition to the thermal radiation effects.

In this paper, the problem of steady MHD boundary layer flow over an inclined plate embedded in a medium which is porous under the influence of linear radiation,  $n^{th}$  order chemical reaction and in the presence of heat generation or absorption effects, Soret and Dofour effects is considered. This paper's objective is to study the impact of heat source parameter, Soret effect, inclination of the plate, Dufour effect and rate of chemical reaction. The dimensionless governing equations of the flow, heat and mass transfer are solved numerically by developing suitable codes in MATLAB's built in solver bvp4c. The numerical results showing the consequences of various values of the material parameters involved in the problem on fluid velocity, heat and mass transfer are displayed by graphs and are then discussed about the effects thoroughly.

## II. PROBLEM FORMULATION

Consider a two dimensional, steady, incompressible, natural convection of a chemically reacting fluid past an inclined plate which has an inclination of an acute angle  $\omega$  with the vertical enclosed in a porous medium. The fluid is also considered to be heat generating or absorbing. In a normal direction to the flow, a uniform magnetic field is applied whose strength is  $B_0$ . The leading edge of the inclined plate is taken as the x-axis and its normal is taken

as the y-axis, i.e. the plate starts at  $x = 0$  which extends towards the x-axis. It is semi infinite in length. Figure 1 shows the physical model of the problem. The free medium is considered to be at temperature  $T_\infty$  and the temperature of plate is maintained uniformly at  $T_W$  such that  $T_W > T_\infty$ . The species concentration at the plate surface is maintained at  $C_W$  and the free stream species concentration is  $C_\infty$  such that  $C_W > C_\infty$ . It is assumed that the steady flow is in the direction of the plate.  $U_\infty$  is taken as the velocity of the flow in free stream. Species concentration and temperature differences cause the variations in density which leads to the actions of buoyancy forces. This as a result starts the convective flow. The magnetic Reynolds number and induced magnetic field are considered to be infinitely very small. It is assumed that the chemical reaction taking place in the flow is homogeneous and is of order  $n$ . The Soret and Dufour effects along with the viscous dissipation effects have been taken into account.

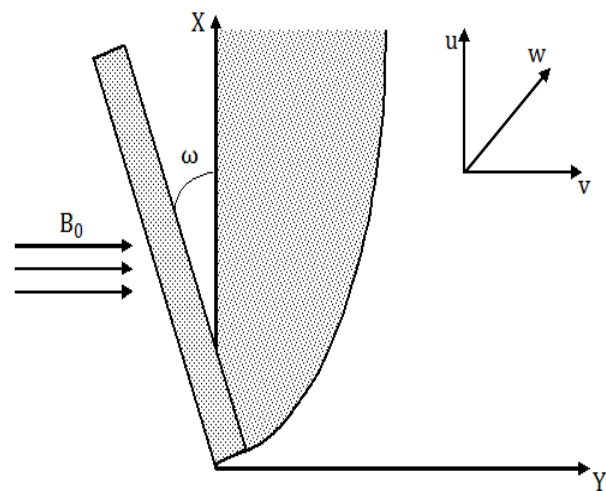


Fig. 1. Physical model of the problem

Then, under the Boussinesq approximations, the equations of continuity, motion, energy and diffusion in cartesian coordinate system are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta_T(T - T_\infty)\cos\omega + g\beta_C(C - C_\infty)\cos\omega - \nu ku - U_\infty - \sigma B_0^2 u \nu, \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\nu}{\lambda C_p} u^2 + \frac{D_m K_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2} + \frac{Q_0}{\rho C_p} (T - T_\infty) - \frac{1}{\rho C_p} \left( \frac{\partial q_r}{\partial y} \right), \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_m K_T}{T_m} \frac{\partial^2 T}{\partial y^2} - K_1 (C - C_\infty)^n. \tag{4}$$

The boundary conditions for this problem are

$$\begin{aligned} v = 0, u = 0, C = C_W, T = T_W \quad \text{at} \quad y = 0, \\ u = U_\infty, C = C_\infty, T = T_\infty \quad \text{as} \quad y \rightarrow \infty. \end{aligned} \tag{5}$$

Introduce following similarity transformations:

$$\begin{aligned} \psi = \sqrt{\nu U_\infty x} f(\eta), \quad \eta = y \sqrt{\frac{U_\infty}{\nu x}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_W - T_\infty}, \\ \phi(\eta) = \frac{C - C_\infty}{C_W - C_\infty}, \end{aligned} \tag{6}$$

where  $\psi(x, y)$  is the stream function and is defined as

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}.$$

The radiative heat flux is denoted by  $q_r$  and is given by using Rosseland's approximation as

$$q_r = -\frac{4\sigma^* \partial T^4}{3k^* \partial y} \quad (7)$$

The equation (7) can be linearized by expanding  $T^4$  about  $T_\infty$  using Taylor series which after neglecting the higher order terms takes the form

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (8)$$

Using relations (6), (7) and (8) into the equations (2) – (4), the following governing equations are obtained:

$$f'''' + \frac{1}{2} f f'' + G_T \cos \omega \theta + G_C \cos \omega \phi - f'(M + Da) + Da = 0, \quad (9)$$

$$\frac{1}{Pr} (1 + R) \theta'' + \frac{1}{2} f \theta' + Ec (f'')^2 + N (f')^2 + D_f \phi'' + \delta \theta = 0, \quad (10)$$

$$\frac{1}{Sc} \phi'' + \frac{1}{2} f \phi' - \gamma \phi^n + Sr \theta'' = 0. \quad (11)$$

The dimensionless parameters which are involved in (9) – (11) are defined as

$$G_T = \frac{g \beta_T (T_W - T_\infty) x}{U_\infty^2}, G_C = \frac{g \beta_C (C_W - C_\infty) x}{U_\infty^2}, M = \frac{\sigma B_0^2 x}{\rho U_\infty},$$

$$Da = \frac{v x}{\kappa U_\infty}, Pr = \frac{\nu}{\alpha}, R = \frac{16 \sigma^* T_\infty^3}{3 \lambda k^*}, Ec = \frac{U_\infty^2}{C_p (T_W - T_\infty)},$$

$$N = \frac{x v U_\infty}{\lambda C_p (T_W - T_\infty)}, \delta = \frac{Q_0 x}{\rho C_p U_\infty}, D_f = \frac{D_m K_T (C_W - C_\infty)}{C_S C_p \nu (T_W - T_\infty)},$$

$$Sc = \frac{\nu}{D_m}, Sr = \frac{D_m K_T (T_W - T_\infty)}{T_m \nu (C_W - C_\infty)}, \gamma = \frac{K_1 x}{U_\infty}. \quad (12)$$

The transformed boundary conditions (5) are

$$f' = 0, f = 0, \phi = 1, \theta = 1 \quad \text{at} \quad \eta = 0$$

$$f' = 1, \phi = 0, \theta = 0 \quad \text{as} \quad \eta \rightarrow \infty. \quad (13)$$

The ordinary differential equations (9) - (11) together with the boundary conditions (13) are non-linear and highly coupled. These equations are therefore solved numerically using *bvp4c* which is the built in solver of MATLAB.

### III. RESULTS AND DISCUSSION

The investigation of steady heat transfer and mass transfer by magnetohydrodynamic boundary flow over an inclined plate in a porous media under the influence of viscous dissipation, linear thermal radiation and chemical reaction of  $n^{\text{th}}$  order and in the presence of heat generation or absorption effects, Soret and Dofour effects is carried out. Numerical calculations are carried out from the solutions for  $f'$ ,  $\theta$ ,  $\phi$  which are the non – dimensional velocity field, temperature field, concentration field by assuming some arbitrary chosen specific values to the physical parameters like  $D_f$ ,  $Sr$ ,  $R$ ,  $\delta$ ,  $n$ ,  $\gamma$  etc. involved in the problem and are plotted against  $\eta$  in Figures 2 - 22.

From the figures 2 – 22, it is clear that for all the parameters with the increase in similarity variable  $\eta$  the binary fluid mixture temperature increases gradually from the inclined plate to maximum value near the plate and then decreases monotonically towards the end of the boundary layer. The concentration of the rarer and lighter components of the fluid mixture decreases monotonically from the extreme value at the inclined plate to its lowest value towards the end of boundary layer i.e. more particles get accumulated at the plate thereby throwing away the lighter particles towards the end of the boundary layer, so that, they can be easily separated. Also the velocity of the fluid increases from the minimum value to its maximum value towards the end of boundary layer similarly as a cubic polynomial function curve.

Figures 2 – 13 are plotted to analyse the velocity, temperature and concentration of species of the fluid for the

variation of  $\delta$ ,  $D_f$ ,  $\gamma$ , and  $Sr$ . The velocity and temperature profiles are found to be increased whereas the species concentration are found to be decreased within the boundary layer with an increase in heat source parameter  $\delta$ , Dufour number  $D_f$ , chemical reaction parameter  $\gamma$  and Soret number  $Sr$ . Since the Soret number and viscosity of the fluid are inversely related therefore the fluid becomes less viscous with the increase in the values of Soret number. Decrement in viscosity makes the fluid less resistive to flow thereby increasing the velocity of the fluid. As viscosity of the fluid decreases, the intermolecular cohesion within the fluid also decreases which thereby heats up the fluid making the species of the fluid less concentrated.

The distributions of temperature, velocity and species concentration of the fluid mixture for several values of the parameter of radiation and order of the chemical reaction are displayed in figures 14 – 19. It is noticed that, both the temperature and fluid velocity decrease while the concentration increases with the increase in the values of radiation parameter  $R$  and order of the chemical reaction  $n$ . The radiation parameter is inversely proportional to the thermal conductivity of the fluid. In general, the thermal conductivity of gases increases with increasing temperature, due to which the temperature of the fluid reduces with the rise in radiation. The thermal conductivity is directly related to the density, the mean molecular speed, and especially to the mean free path of molecules of the fluid mixture. Therefore the rarer and lighter components within the fluid get denser with the low thermal conductivity thereby increasing the species concentration of the fluid. Also introducing more thermal radiation reduces the thermal conductivity of the fluid which as a result leads to low velocity of the fluid. Rate of reaction is the speed at which a chemical reaction occurs. In a reaction with low rate the molecules combine at a slower speed as compared to a reaction with a high rate. Increase in rate of reaction will complete the reaction faster which as a result will enhance the possible combinations of molecules bouncing into each other i.e. more collisions occur in the system thereby intensifying the species concentration.

The effects of the inclination of the plate on the fluid flow, heat and mass transfer are shown in figures 20 – 22. From the figures it is evident that the fluid velocity decreases whereas the fluid temperature and species concentration enhance with the inclination of the plate.

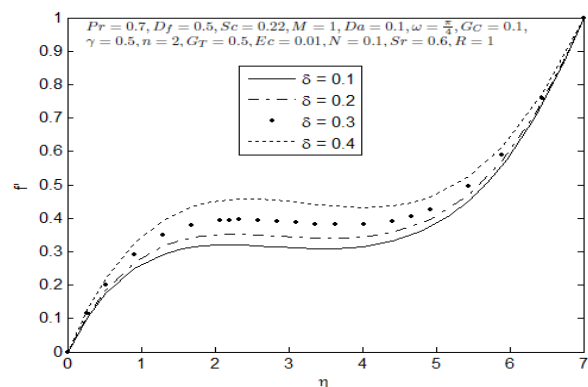


Fig. 2. Profile of velocity for  $\delta$

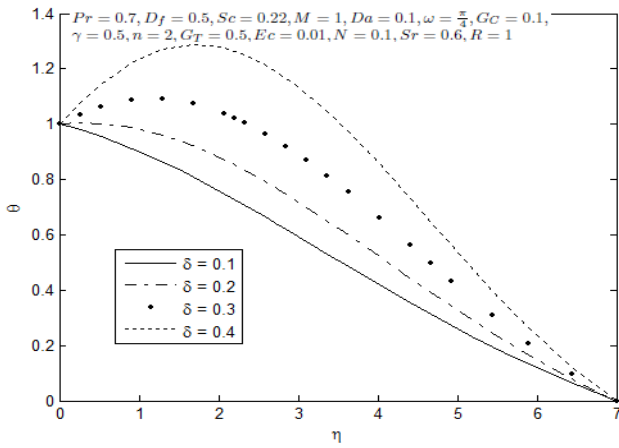


Fig. 3. Profile of temperature for  $\delta$

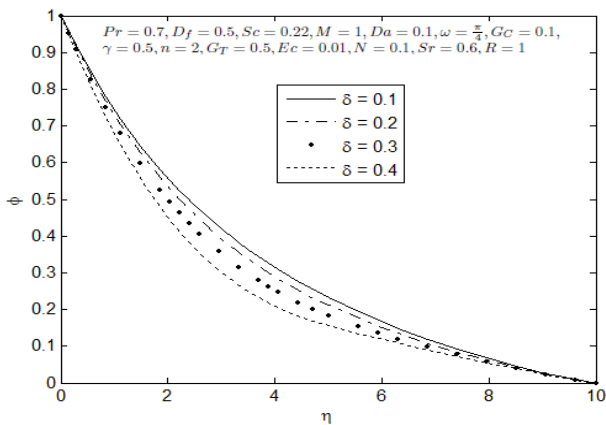


Fig. 4. Profile of concentration for  $\delta$

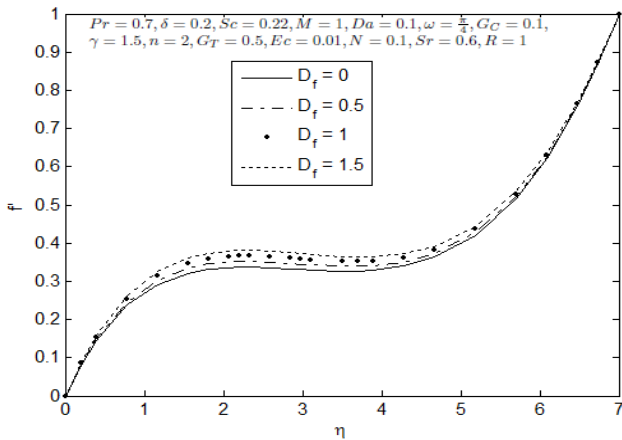


Fig. 5. Profile of velocity for  $D_f$

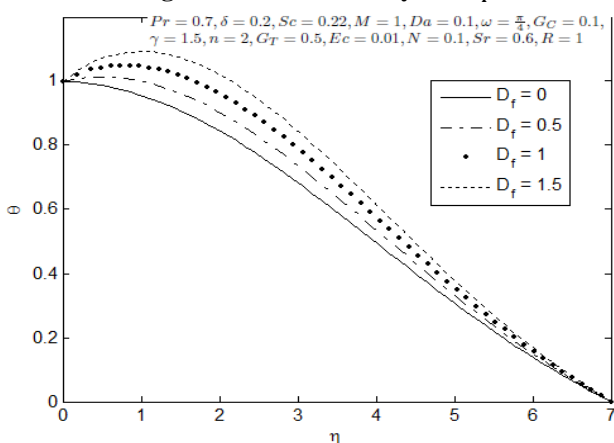


Fig. 6. Profile of temperature for  $D_f$

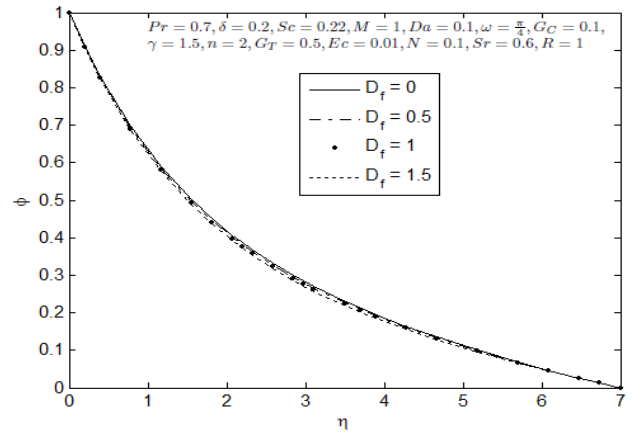


Fig. 7. Profile of concentration for  $D_f$

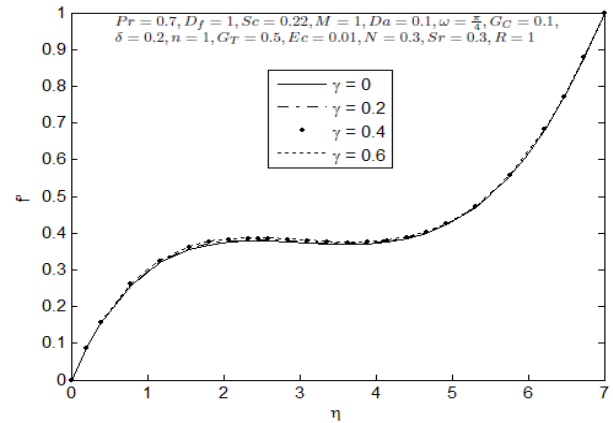


Fig. 8. Profile of velocity for  $\gamma$

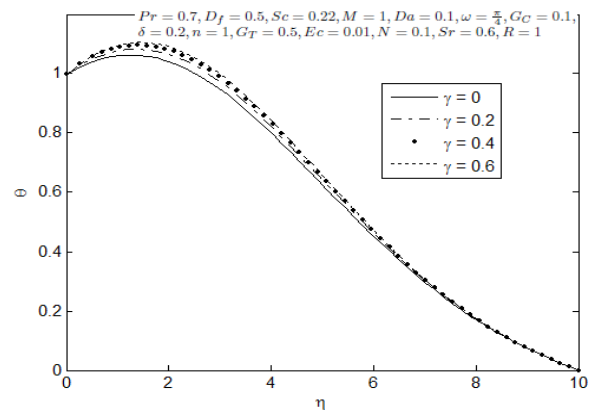


Fig. 9. Profile of temperature for  $\gamma$

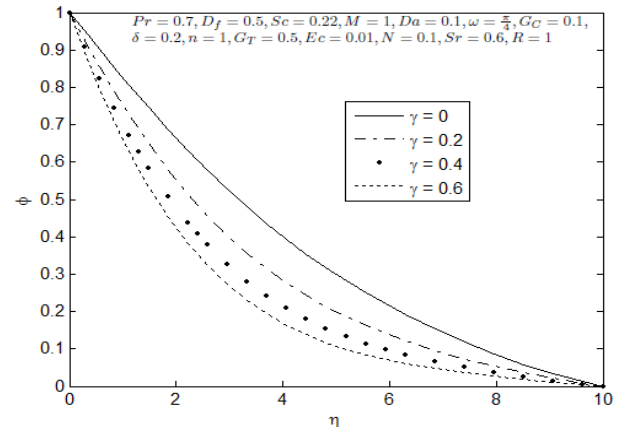


Fig. 10. Profile of concentration for  $\gamma$

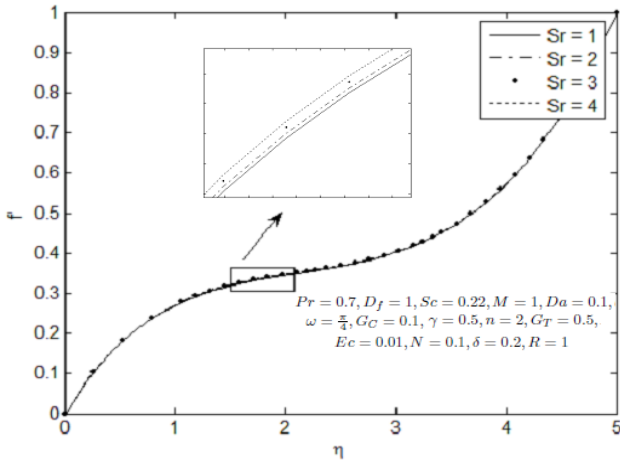


Fig. 11. Profile of velocity for  $Sr$

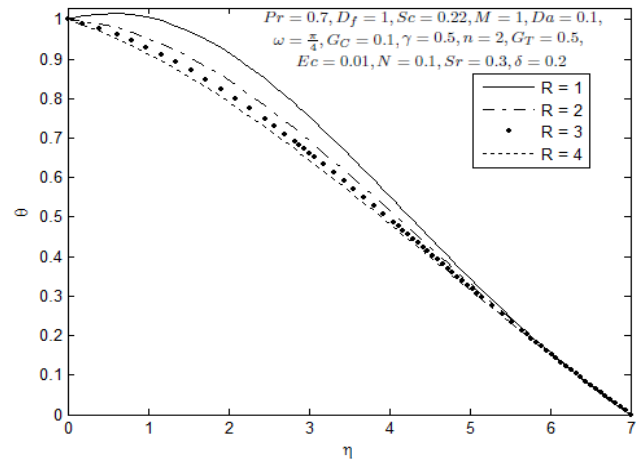


Fig. 15. Profile of temperature for  $R$

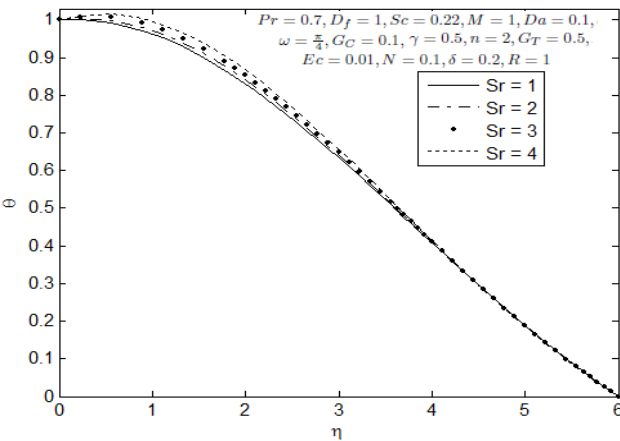


Fig. 12. Profile of temperature for  $Sr$

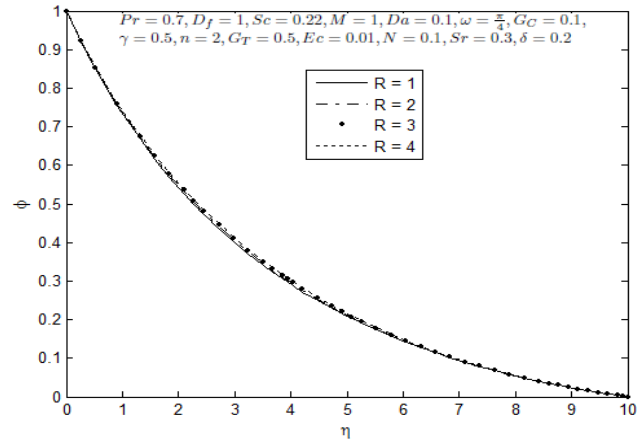


Fig. 16. Profile of concentration for  $R$

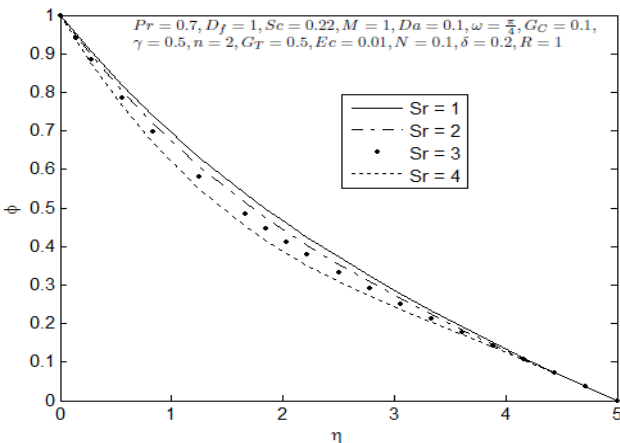


Fig. 13. Profile of concentration for  $Sr$

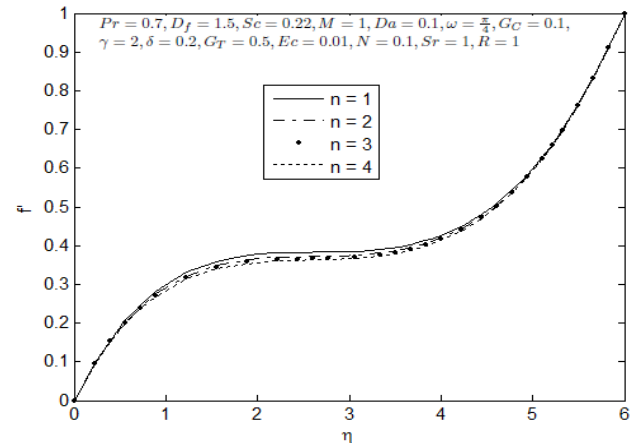


Fig. 17. Profile of velocity for  $n$

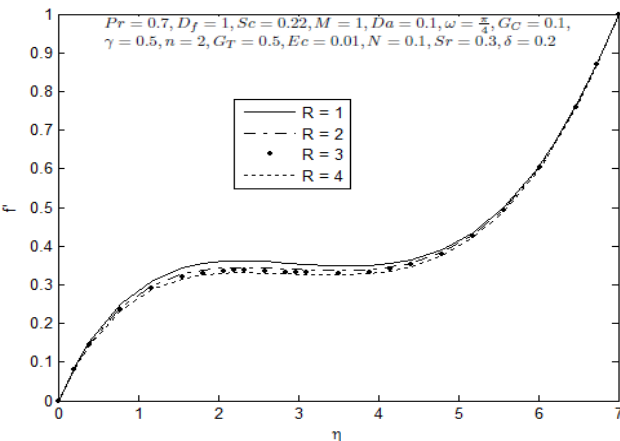


Fig. 14. Profile of velocity for  $R$

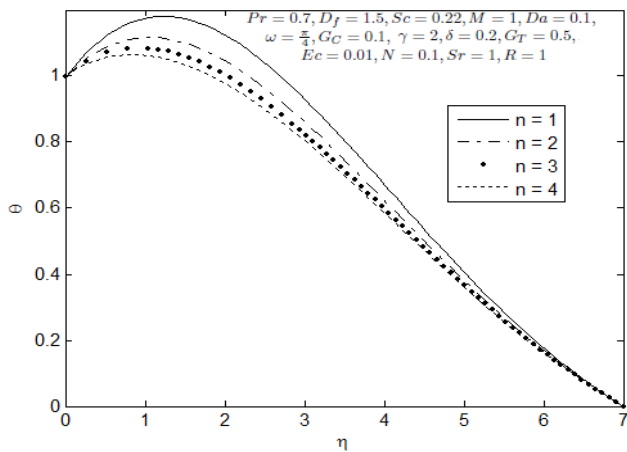


Fig. 18. Profile of temperature for  $n$

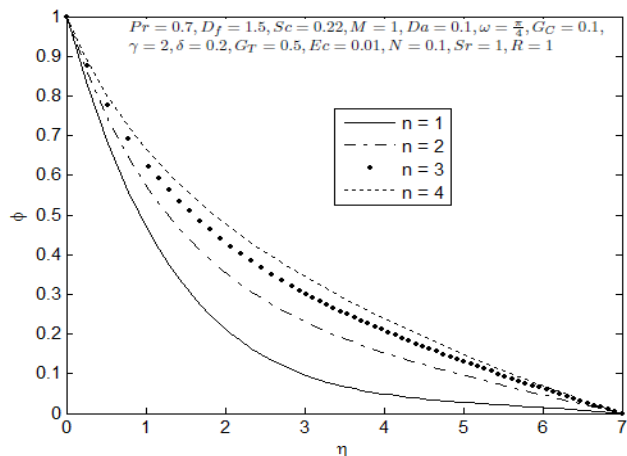


Fig. 19. Profile of concentration for  $n$

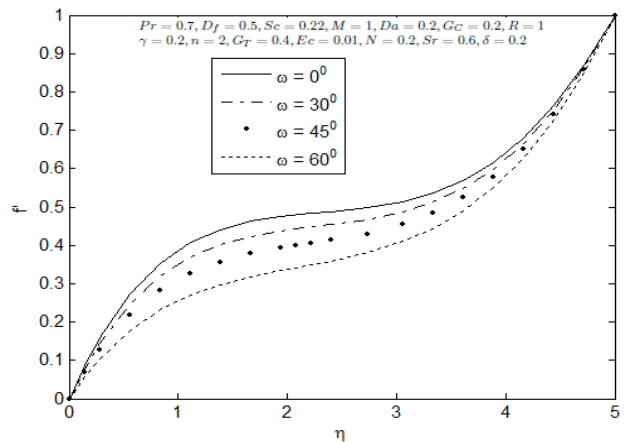


Fig. 20. Profile of velocity for  $\omega$

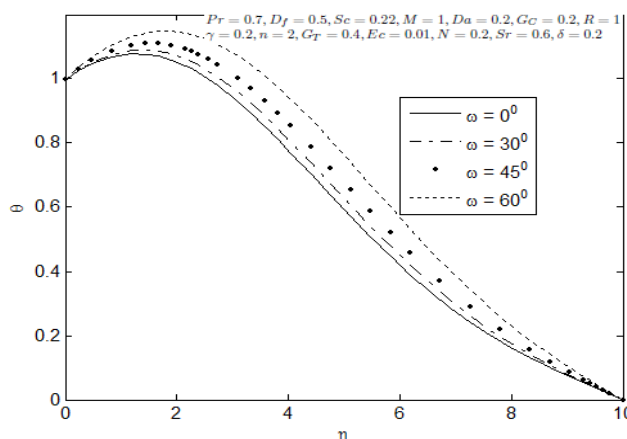


Fig. 21. Profile of temperature for  $\omega$

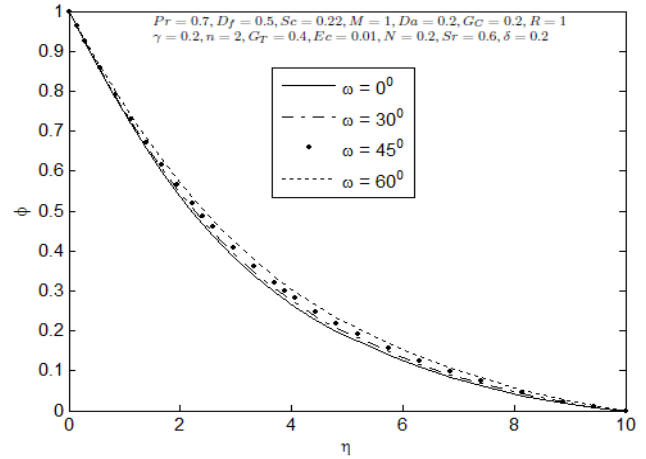


Fig. 22. Profile of concentration for  $\omega$

#### IV. CONCLUSION

Significant findings of the present problem are mentioned below:

- Rise in the heat generation / absorption effect, Dufour effect, Soret effect and chemical reaction lead to the heating up of the binary fluid mixture and the process of demixing the components of the fluid mixture is enhanced.
- The rate of reaction and thermal radiation make the binary fluid mixture colder and slow down the flow of the fluid motion by making the fluid more concentrated.
- The inclination of the plate slows down the fluid motion and heats up the fluid which leads the species present in the fluid to become denser.
- All the material parameters collect the heavier particles near the inclined plate thereby throwing away the lighter particles away from the plate towards the end of the boundary layer which as a result encourages the demixing of the fluid mixture.

Numerical results are obtained in this paper so it is yet to be tested experimentally the results so obtained in this paper. As the cases taken up are based on a simple geometry hence it will be easy to test these results experimentally.

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