



# Fuzzy Positive Implicative and Fuzzy Associative WI-Ideals of Lattice Wajsberg Algebras

C. Shajitha Begum, A. Ibrahim

**Abstract:** We introduce the definitions of fuzzy positive implicative WI-ideal and fuzzy associative WI-ideal of lattice Wajsberg algebra. Further, we prove every fuzzy positive implicative WI-ideal of lattice Wajsberg algebra is a fuzzy WI-ideal. Also, we discuss the relationship between fuzzy positive implicative WI-ideal and level subsets. Moreover, we give the relationship of fuzzy associative WI-ideal with fuzzy WI-ideal in lattice Wajsberg algebra. Finally, we study some characterizations of fuzzy associative WI-ideal.

**Keywords:** Wajsberg algebra; Lattice Wajsberg algebra; WI-ideal; Implicative WI-ideal; Positive implicative WI-ideal; Associative WI-ideal; Fuzzy positive implicative WI-ideal; Fuzzy associative WI-ideal.  
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## I. INTRODUCTION

The term fuzzy logic was introduced by Zadeh[9] in 1965. Fuzzy logic has been applied to many fields, from control theory to artificial intelligence. The concept of Wajsberg algebra was first proposed by MordchajWajsberg[8]in 1935, and analyzed by Font, Rodriguez, and Torrens[1] in 1984. They [2] also introduced a lattice structure of Wajsberg algebra. The authors [2] introduced the notion of WI-ideal of lattice Wajsberg algebra and discussed some related properties. Further, the authors [3], [4], [5], [6], “to be published”[7]introduced the notions of fuzzy WI-ideal, normal fuzzy WI-ideal, implicative WI-ideal, fuzzy implicative WI-ideal, an anti fuzzy WI-ideal ,positive implicative WI-ideal, associative WI-ideal of lattice Wajsberg

algebras and also investigated their properties with suitable illustrations.

In this paper, we introduce the definitions of fuzzy positive implicative WI-ideal and fuzzy associative WI-ideal of lattice Wajsberg algebra. Also, we discuss the relationship between fuzzy positive implicative WI-ideal and fuzzy WI-ideal. Further, we prove that every fuzzy associative WI-ideal of lattice Wajsberg algebra with respect to 0 is a fuzzy WI-ideal. Finally, we give some of the characterizations of fuzzy associative WI-ideal.

## II. PRELIMINARIES

We recollect basic definitions and their properties which are useful to develop our main results.

**Definition 2.1[1].** Let  $(A, \rightarrow, *, 1)$  be an algebra with binary operation ‘ $\rightarrow$ ’ and a quasi complement ‘ $*$ ’ is said to be Wajsberg algebra if it satisfies the following,

- (i)  $1 \rightarrow x = x$ ;
- (ii)  $(x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$ ;
- (iii)  $(x \rightarrow y) \rightarrow ((y \rightarrow z) \rightarrow (x \rightarrow z)) = 1$ ;
- (iv)  $(x^* \rightarrow y^*) \rightarrow (y \rightarrow x) = 1$  for all  $x, y, z \in A$ .

**Proposition 2.2[1].** A Wajsberg algebra  $(A, \rightarrow, *, 1)$  satisfies the following axioms for all  $x, y, z \in A$ ,

- (i)  $x \rightarrow x = 1$ ;
- (ii) If  $(x \rightarrow y) = (y \rightarrow x) = 1$  then  $x = y$ ;
- (iii) If  $(x \rightarrow y) = (y \rightarrow z) = 1$  then  $x \rightarrow z = 1$ ;
- (iv)  $(x \rightarrow (y \rightarrow x)) = 1$ ;
- (v)  $(x \rightarrow y) \rightarrow ((z \rightarrow x) \rightarrow (z \rightarrow y)) = 1$ ;
- (vi)  $x \rightarrow 1 = 1$ ;
- (vii)  $(x^*)^* = x$ ;
- (viii)  $(x^* \rightarrow y^*) = y \rightarrow x$ ;
- (ix)  $x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$ ;
- (x)  $x \rightarrow 0 = x \rightarrow 1^* = x^*$ .

**Definition 2.3[1].** A Wajsberg algebra  $(A, \rightarrow, *, 1)$  is said to be a lattice Wajsberg algebra if it satisfies the following axioms,

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- (i) The partial ordering ‘ $\leq$ ’ on a lattice Wajsberg algebra  $A$ , such that  $x \leq y$  if and only if  $x \rightarrow y = 1$ ;
- (ii)  $x \wedge y = ((x^* \rightarrow y^*) \rightarrow y)^*$ ;
- (iii)  $x \vee y = ((x \rightarrow y) \rightarrow y)$  for all  $x, y \in A$ .

**Note.** From definition 2.3 an algebra  $(A, \vee, \wedge, *, 0, 1)$  is a lattice Wajsberg algebra with lower bound zero and upper bound one.

**Definition 2.4[2].** A nonempty subset  $I$  of lattice Wajsberg algebra  $A$  is said to be *WI-ideal* of lattice Wajsberg algebra  $A$ . If it satisfies,

- (i)  $0 \in I$ ;
- (ii)  $(x \rightarrow y)^* \in I$  and  $y \in I$  imply  $x \in I$  for all  $x, y \in A$ .

**Definition 2.5[7].** Let  $I$  be a non-empty subset of lattice Wajsberg algebra  $A$ .  $I$  is said to be a positive implicative *WI-ideal* of  $A$ , if it satisfies for all  $x, y, z \in A$ ,

- (i)  $0 \in I$ ;
- (ii)  $x \in I$  and  $((y \rightarrow (z \rightarrow y)^*)^* \rightarrow x)^* \in I$  imply  $y \in I$ .

**Definition 2.6[7].** A non-empty subset  $I$  of  $A$  is called an associative *WI-ideal* of  $A$  with respect to  $x$ , where  $x$  is a fixed element of  $A$ , if it satisfies for all  $x, y, z \in A$ ,

- (i)  $0 \in I$ ;
- (ii)  $((z \rightarrow y)^* \rightarrow x^*) \in I$  and  $(y \rightarrow x)^* \in I$  imply  $z \in I$  and  $x \neq 1$ .

**Definition 2.7[9].** Let  $A$  be a set. A function  $\mu : A \rightarrow [0, 1]$  is said to be a fuzzy subset on  $A$ , for each  $x \in A$  the value of  $\mu(x)$  denotes a degree of membership of  $x$  in  $\mu$ .

**Definition 2.8[9].** Let  $\mu$  be a fuzzy subset in a set  $A$ . Then the set  $\{\mu_t = x \in A / \mu(x) \geq t\}$  for  $t \in [0, 1]$  is called level subset of  $\mu$ .

**Definition 2.9[3].** Let  $A$  be a lattice Wajsberg algebra. A fuzzy subset  $\mu$  of  $A$  is said to be a fuzzy *WI-ideal* of lattice Wajsberg algebra  $A$  if,

- (i)  $\mu(0) \geq \mu(x)$ ;
- (ii)  $\mu(x) \geq \min \{ \mu(x \rightarrow y)^*, \mu(y) \}$  for all  $x, y \in A$ .

**Proposition 2.10[7].** Let  $M$  and  $N$  be two *WI-ideals* of lattice Wajsberg algebra  $A$  with  $M \subseteq N$ . If  $M$  is a positive implicative *WI-ideal* of  $A$  then so is  $N$ .

### III. MAIN RESULTS

#### 3.1. Fuzzy Positive Implicative *WI-ideal*

We introduce the concept of fuzzy positive implicative *WI-ideal* of lattice Wajsberg algebra  $A$  and study some of its properties.

**Definition 3.1.1.** A fuzzy subset  $\mu$  of lattice Wajsberg algebra  $A$  is called a fuzzy positive implicative *WI-ideal* of  $A$  if for all  $x, y, z \in A$ ,

- (i)  $\mu(0) \geq \mu(x)$ ;
- (ii)  $\mu(y) \geq \min \{ \mu(((y \rightarrow (z \rightarrow y)^*)^* \rightarrow x)^*), \mu(x) \}$ .

**Example 3.1.2.** A set  $A = \{0, i, j, k, 1\}$  with partial ordering as in the ‘Fig.1’. Defining a binary operation ‘ $\rightarrow$ ’ and a quasi complement ‘ $*$ ’ on  $A$  as given in tables I and II.




Fig. 1  
Lattice diagram

**Table I: Complement**

$x$	$x^*$
0	1
$i$	$k$
$j$	$j$
$k$	$i$
1	0

**Table II: Implication**

$\rightarrow$	0	$i$	$j$	$k$	1
0	1	1	1	1	1
$i$	$k$	1	1	1	1
$j$	$j$	$k$	1	1	1
$k$	$i$	$j$	1	1	1
1	0	$i$	$j$	$k$	1

Define  $\wedge$  and  $\vee$  on  $A$  as follows:

$$x \wedge y = ((x^* \rightarrow y^*) \rightarrow y)^*; \quad x \vee y = ((x \rightarrow y) \rightarrow y) \text{ for all } x, y \in A.$$

Then,  $(A, \vee, \wedge, \rightarrow, 0, 1)$  is a lattice Wajsberg algebra. A fuzzy subset  $\mu$  of  $A$  is defined by,

$$\mu(x) = \begin{cases} .68 & \text{when } x = 0 \\ .21 & \text{when } x = \{i, j, k, 1\} \end{cases} \text{ for all } x \in A$$

Then, a fuzzy subset  $\mu$  is a fuzzy positive implicative *WI-ideal* of lattice Wajsberg algebra  $A$ .

**Proposition 3.1.3.** Every fuzzy positive implicative *WI-ideal* of lattice Wajsberg algebra  $A$  is a fuzzy *WI-ideal* of  $A$ .

**Proof.** Let  $\mu$  be a fuzzy positive implicative *WI-ideal* of  $A$ , then from (ii) of definition 3.1.1 we have

$$\mu(y) \geq \min \{ \mu(((y \rightarrow (z \rightarrow y)^*)^* \rightarrow x)^*), \mu(x) \} \text{ for all } x, y, z \in A. \tag{3.1.1}$$

Taking  $x = y, y = x$  and  $z = x$  in (3.1.1)

We obtain

$$\mu(x) \geq \min \{ \mu(x \rightarrow (x \rightarrow x)^* \rightarrow y)^*, \mu(y) \}$$

$$\begin{aligned} &= \min \{ \mu ( ((x \rightarrow 1)^* \rightarrow y)^* ), \mu(y) \} \\ &= \min \{ \mu ( ((x \rightarrow 0)^* \rightarrow y)^* ), \mu(y) \} \\ &= \min \{ \mu ( (x \rightarrow y)^* ), \mu(y) \}. \end{aligned}$$

Thus, we have  $\mu(x) \geq \min \{ \mu ( (x \rightarrow y)^* ), \mu(y) \}$ , and  $\mu(0) \geq \mu(x)$  [From (i) of definition 3.1.1]

Hence,  $\mu$  is a fuzzy *WI*-ideal of  $A$ . ■

**Note.** The converse of the above proposition may not be true.

**Proposition 3.1.4.** Let  $\mu$  be a fuzzy implicative *WI*-ideal of lattice Wajsberg algebra  $A$ .  $\mu$  is a fuzzy positive implicative *WI*-ideal of  $A$  if and only if  $\mu(x) \geq \mu( ((x \rightarrow (y \rightarrow x)^*)^* ))$  for all  $x, y \in A$ .

**Proof.** Let  $\mu$  be a fuzzy positive implicative *WI*-ideal of  $A$ , then from (ii) of definition 3.1.1 we have

$$\mu(y) \geq \min \{ \mu ( ((y \rightarrow (z \rightarrow y)^*)^* \rightarrow x)^* ), \mu(x) \} \quad \text{for all } x, y, z \in A. \quad (3.1.2)$$

Substituting  $x = 0, y = x$ , and  $z = y$  in (3.1.2)

$$\begin{aligned} \text{We obtain } \mu(x) &\geq \min \{ \mu ( ((x \rightarrow (y \rightarrow x)^*)^* \rightarrow 0)^* ), \mu(0) \} \\ &= \min \{ \mu ( (x \rightarrow (y \rightarrow x)^* ) ), \mu(0) \} \\ &= \mu ( (x \rightarrow (y \rightarrow x)^* ) ) \end{aligned}$$

Conversely, suppose  $\mu$  is a fuzzy *WI*-ideal and it satisfies the inequality,

$$\mu(x) \geq \mu ( (x \rightarrow (y \rightarrow x)^* ) ) \quad \text{for all } x, y, z \in A \quad (3.1.3)$$

Put  $x = y$  in (3.1.3), then, we have

$$\begin{aligned} \mu(y) &\geq \mu ( (y \rightarrow (z \rightarrow y)^* ) ) \\ &\geq \min \{ \mu ( ((y \rightarrow (z \rightarrow y)^*)^* \rightarrow x)^* ), \mu(x) \}. \end{aligned}$$

Thus, we have

$$\mu(y) \geq \min \{ \mu ( ((y \rightarrow (z \rightarrow y)^*)^* \rightarrow x)^* ), \mu(x) \},$$

and  $\mu(0) \geq \mu(x)$  [From (i) of definition 3.1.1]

Hence,  $\mu$  is a fuzzy positive implicative *WI*-ideal of  $A$ . ■

**Proposition 3.1.5.** If  $\mu$  is a fuzzy positive implicative *WI*-ideal of lattice Wajsberg algebra  $A$  then,  $I = \{ x \in A / \mu(x) = \mu(0) \}$  is a positive implicative *WI*-ideal of  $A$ .

**Proof.** Let  $\mu$  be a fuzzy positive implicative *WI*-ideal of  $A$  and  $I = \{ x \in A / \mu(x) = \mu(0) \}$ .

Obviously,  $0 \in A$ . Let  $((y \rightarrow (z \rightarrow y)^*)^* \rightarrow x)^* \in I, x \in I$  for all  $x, y, z \in A$ .

$$\begin{aligned} \text{Then, we have } \mu ( ((y \rightarrow (z \rightarrow y)^*)^* \rightarrow x)^* ) &= \mu(0) \text{ and} \\ \mu(x) &= \mu(0) \end{aligned} \quad (3.1.4)$$

Since  $\mu$  is a fuzzy positive implicative *WI*-ideal, we have  $\mu(y) \geq \min \{ \mu ( ((y \rightarrow (z \rightarrow y)^*)^* \rightarrow x)^* ), \mu(x) \}$

[From (ii) of definition 3.1.1]

$$= \mu(0) \quad \text{[From 3.1.4]}$$

and  $\mu(0) \geq \mu(x)$  [From (i) of definition 3.1.1]

Then, we get  $\mu(y) = \mu(0)$

Thus,  $y \in I$  it follows that  $I$  is a positive implicative *WI*-ideal of  $A$ . ■

**Proposition 3.1.6.** Let  $\mu$  be a fuzzy subset of lattice Wajsberg algebra  $A$ .  $\mu$  is a fuzzy positive implicative *WI*-ideal of  $A$  if and only if  $\mu(\alpha, \beta)$  is a positive implicative *WI*-ideal of  $A$ , when  $\mu(\alpha, \beta) \neq \emptyset; \alpha, \beta \in [0, 1]$ .

**Proof.** Let  $\mu$  be a fuzzy positive implicative *WI*-ideal of  $A$  and  $\alpha, \beta \in [0, 1]$  such that  $\mu(\alpha, \beta) \neq \emptyset$ . Clearly  $0 \in \mu(\alpha, \beta)$ .

Let  $((y \rightarrow (z \rightarrow y)^*)^* \rightarrow x)^* \in \mu(\alpha, \beta), x \in \mu(\alpha, \beta)$  for all  $x, y, z \in A$ . Then, we have

$$\mu ( ((y \rightarrow (z \rightarrow y)^*)^* \rightarrow x)^* ) \geq [\alpha, \beta], \mu(x) \geq [\alpha, \beta].$$

It follows that,

$$\mu(y) \geq \min \{ \mu ( ((y \rightarrow (z \rightarrow y)^*)^* \rightarrow x)^* ), \mu(x) \} \geq [\alpha, \beta].$$

Thus,  $y \in \mu[\alpha, \beta]$ . Hence, we have  $\mu[\alpha, \beta]$  is a positive implicative *WI*-ideal of  $A$ .

Conversely, suppose that  $\mu(\alpha, \beta) \neq \emptyset$  is a positive implicative *WI*-ideal of  $A$ , where  $\alpha, \beta \in [0, 1]$ . For any  $x \in A$ , and  $x \in \mu_\mu(x)$ , it follows that  $\mu_\mu(x)$  is a positive implicative *WI*-ideal of  $A$ .

Thus,  $0 \in \mu_\mu(x)$ . That is  $\mu(0) \geq \mu(x)$  for all  $x, y, z \in A$ .

Let  $[\alpha, \beta] = \min \{ \mu ( ((y \rightarrow (z \rightarrow y)^*)^* \rightarrow x)^* ), \mu(x) \}$ , it follows that  $\mu(\alpha, \beta)$  is a positive implicative *WI*-ideal and  $((y \rightarrow (z \rightarrow y)^*)^* \rightarrow x)^* \in \mu[\alpha, \beta]$ , and  $x \in \mu[\alpha, \beta]$ .

This implies that  $y \in \mu[\alpha, \beta]$ . So,

$$\mu(y) \geq [\alpha, \beta] = \min \{ \mu ( ((y \rightarrow (z \rightarrow y)^*)^* \rightarrow x)^* ), \mu(x) \}$$

Hence, we get  $\mu$  is a fuzzy positive implicative *WI*-ideal of  $A$ . ■

**Corollary 3.1.7.** A fuzzy subset  $\mu$  of lattice Wajsberg algebra  $A$  is a fuzzy positive implicative *WI*-ideal of  $A$  if and only if  $\mu_\alpha$  is a positive implicative *WI*-ideal of  $A$ , when  $\mu_\alpha \neq \emptyset, \alpha \in [0, 1]$ .

**Proposition 3.1.8.** Let  $M$  and  $N$  be implicative *WI*-ideals of lattice Wajsberg algebra  $A$ , such that  $M \subseteq N$ . If  $\mu$  is a fuzzy



positive implicative WI-ideal of  $M$ . Then so  $N$ .

**Proof.** Let  $M$  and  $N$  be implicative WI-ideals of lattice Wajsberg algebra  $A$ . Let  $\mu$  be a fuzzy positive implicative WI-ideal of  $M$ . Since  $M \subseteq N$ ,  $\mu_M(x) \leq \mu_N(x)$  for all  $x \in A$ . Then, clearly  $M_\alpha \leq N_\alpha$  for every  $\alpha \in [0,1]$ . If  $\mu_M$  is a fuzzy positive implicative WI-ideal of  $A$ . Hence, we get  $M_\alpha$  is a positive implicative WI-ideal of  $A$ . [From corollary 3.1.7]

Then,  $N_\alpha$  is a positive implicative WI-ideal of  $A$ .  
[From Proposition 2.10]

Thus,  $\mu_N$  is fuzzy positive implicative WI-ideal. Hence,  $\mu$  is a fuzzy positive implicative WI-ideal of  $N$ . ■

**3.2. Fuzzy Associative WI-ideal**

We introduce an idea of fuzzy associative WI-ideal of lattice Wajsberg algebra  $A$  and examine its properties,

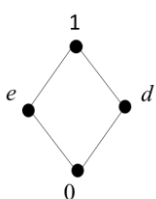
**Definition 3.2.1.** A fuzzy subset  $\mu$  of lattice Wajsberg algebra  $A$  is said to be a fuzzy associative WI-ideal of  $A$  with respect to  $x$ , where  $x$  is a fixed element of  $A$ , if it satisfies,

- (i)  $\mu(0) \geq \mu(x)$ ;
- (ii)  $\mu(z) \geq \min\{\mu((z \rightarrow y)^* \rightarrow x)^*, \mu((y \rightarrow x)^*)\}$  for all  $x, y, z \in A$ .

A fuzzy associative WI-ideal with respect to all  $x \neq 1$  is called a fuzzy associative WI-ideal. Fuzzy associative WI-ideal with respect to 1 is constant.

**Example 3.2.2.** A set  $A = \{0, d, e, 1\}$  with partial ordering as in “Fig. 2”. Define ‘\*’ and ‘ $\rightarrow$ ’ on  $A$  as given in tables III and IV.

Table III: Complement    Table IV: Implication



$x$	$x^*$
0	1
$d$	$e$
$e$	$d$
1	0

$\rightarrow$	0	$d$	$e$	1
0	1	1	1	1
$d$	$e$	1	1	1
$e$	$d$	$e$	1	1
1	0	$d$	$e$	1

**Fig.2. Lattice diagram**

Here,  $A$  is a lattice Wajsberg algebra. A fuzzy subset  $\mu$  of  $A$  is defined by,

$$\mu(x) = \begin{cases} .54 & \text{when } x = 0 \\ .33 & \text{when } x = \{d, e, 1\} \end{cases} \text{ for all } x \in A$$

Then,  $\mu$  is a fuzzy associative WI-ideal of  $A$ .

**Proposition 3.2.3.** If  $\mu$  is a fuzzy associative WI-ideal of  $A$  with respect to  $x$  then  $\mu(x) = \mu(0)$ .

**Proof.** Let  $\mu$  be a fuzzy associative WI-ideal of  $A$  with respect to  $x$  if  $x = (0, 1)$ . Then it is trivial, we assume that  $x$  is neither 0 nor 1. Then,

$$\mu(x) \geq \min\{\mu(((x \rightarrow 0)^* \rightarrow x)^*), \mu((0 \rightarrow x)^*)\}$$

[From (ii) of definition 3.2.1]

Hence, we get  $\mu(x) = \mu(0)$ . ■

**Proposition 3.2.4.** Every fuzzy associative WI-ideal of lattice Wajsberg algebra  $A$  with respect to 0 is a fuzzy WI-ideal of  $A$ .

**Proof.** If  $\mu$  is a fuzzy associative WI-ideal of  $A$  with respect to 0. Then, we have

$$\mu(x) \geq \min\{\mu((x \rightarrow y)^* \rightarrow 0), \mu((y \rightarrow 0)^*)\} \text{ for all } x, y \in A$$

[From (ii) of definition 3.2.1]

$$= \min\{\mu((x \rightarrow y)^*), \mu(y)\}$$

Hence, we have  $\mu$  is a fuzzy WI-ideal of  $A$ . ■

**Proposition 3.2.5.** Let  $\mu$  be a fuzzy WI-ideal of lattice Wajsberg algebra  $A$ .  $\mu$  is a fuzzy associative WI-ideal of  $A$  if and only if it satisfies,  $\mu((z \rightarrow (y \rightarrow x)^*)^*) \geq \mu((z \rightarrow y^* \rightarrow x^*)^*)$  for all  $x, y, z \in A$ .

**Proof.** Let  $\mu$  be a fuzzy WI-ideal of  $A$  satisfying  $\mu((z \rightarrow y^* \rightarrow x^*)^*) \geq \mu(z \rightarrow y^* \rightarrow x^*)$  for all  $x, y, z \in A$

Then we have,

$$\mu(z) \geq \min\{\mu((z \rightarrow (y \rightarrow x)^*)^*), \mu((y \rightarrow x)^*)\}$$

$$= \min\{\mu(((z \rightarrow y)^* \rightarrow x)^*), \mu((y \rightarrow x)^*)\}$$

So,  $\mu$  is a fuzzy associative WI-ideal of  $A$ .

Conversely, suppose that  $\mu$  be a fuzzy associative WI-ideal of  $A$ . Then we have,  $\mu((z \rightarrow (y \rightarrow x)^*)^*) \geq \min\{\mu((z \rightarrow y^* \rightarrow x^*)^*), \mu(z \rightarrow y^* \rightarrow x^*)\}$

Let us consider,

$$\begin{aligned} & (((z \rightarrow (y \rightarrow x)^*)^* \rightarrow (z \rightarrow y)^*)^* \rightarrow x \\ & = x^* \rightarrow ((z \rightarrow (y \rightarrow x)^*)^* \rightarrow (z \rightarrow y)^*) \\ & = x^* \rightarrow ((z \rightarrow y) \rightarrow (z \rightarrow (y \rightarrow x)^*)) \\ & = (x \rightarrow y) \rightarrow (x^* \rightarrow ((y \rightarrow x) \rightarrow z^*)) \\ & = (z \rightarrow y) \rightarrow ((y \rightarrow x) \rightarrow (x^* \rightarrow z^*)) \\ & = (z \rightarrow y) \rightarrow ((z \rightarrow y) \rightarrow (z \rightarrow y)) \\ & = 1 \end{aligned}$$

It follows that



$$\begin{aligned} \mu((z \rightarrow (y \rightarrow x)^*)) &\geq \min\{\mu(0), \mu(((z \rightarrow y)^* \rightarrow x)^*)\} \\ &= \mu(((z \rightarrow y)^* \rightarrow x)^*) \end{aligned}$$

Hence, we have

$$\mu((z \rightarrow (y \rightarrow x)^*)) \geq \mu(((z \rightarrow y)^* \rightarrow x)^*). \blacksquare$$

**Proposition 3.2.6.** Let  $\mu$  be a fuzzy WI-ideal of lattice Wajsberg algebra  $A$ .  $\mu$  is a fuzzy associative WI-ideal of  $A$  if and only if it satisfies  $\mu(z) \geq \mu(((z \rightarrow x) \rightarrow x)^*)$  for all  $x, z \in A$ .

**Proof.** Let  $\mu$  be a fuzzy associative WI-ideal of  $A$ .

Then from (ii) of definition 3.2.1 we have,

$$\mu(z) \geq \min\{\mu(((z \rightarrow y)^* \rightarrow x)^*), \mu((y \rightarrow x)^*)\} \quad \text{for all } x, y, z \in A. \quad (3.2.1)$$

Taking  $y = x$  in 3.2.1 we get,

$$\begin{aligned} \mu(z) &\geq \min\{\mu(((z \rightarrow x)^* \rightarrow x)^*), \mu((x \rightarrow x)^*)\} \\ &= \min\{\mu(((z \rightarrow x)^* \rightarrow x)^*), \mu(0)\} \\ &= \mu(((z \rightarrow x)^* \rightarrow x)^*) \end{aligned}$$

Conversely, if  $\mu$  is a fuzzy WI-ideal and satisfies  $\mu(z) \geq \mu(((z \rightarrow x) \rightarrow x)^*)$  for  $x, z \in A$

$$\text{Clearly, } (((z \rightarrow x)^* \rightarrow (y \rightarrow x)^*) \rightarrow (z \rightarrow y)^*)^* = 0$$

$$\text{and } ((z \rightarrow y)^* \rightarrow (z \rightarrow x)^*)^* \leq (x \rightarrow y)^*$$

It follows that,

$$\begin{aligned} (((z \rightarrow (y \rightarrow x)^*)^* \rightarrow x)^* \rightarrow x)^* \rightarrow ((z \rightarrow y)^* \rightarrow x)^*)^* &= 0 \\ \mu((z \rightarrow (y \rightarrow x)^*)^*) &\geq \mu(((z \rightarrow y \rightarrow x)^* \rightarrow x)^* \rightarrow x)^*) \\ &\geq \min\{\mu(((z \rightarrow (y \rightarrow x)^* \rightarrow x)^*)^*), \mu((z \rightarrow y)^* \rightarrow x)^*)\} \end{aligned}$$

$$\begin{aligned} \mu(((z \rightarrow y)^* \rightarrow x)^*) &= \min\{\mu(0), \mu(((z \rightarrow y)^* \rightarrow x)^*)\} \\ &= \mu(((z \rightarrow y)^* \rightarrow x)^*) \end{aligned}$$

From proposition 3.2.3, we get  $\mu$  is a fuzzy associative WI-ideal of  $A$ .  $\blacksquare$

#### IV. CONCLUSION

In this paper, we have introduced the notions of fuzzy positive implicative WI-ideal and fuzzy associative WI-ideal of lattice Wajsberg algebras. Further, we have discussed the relationship between fuzzy positive implicative WI-ideal and fuzzy WI-ideal, fuzzy associative WI-ideal and fuzzy WI-ideal in lattice Wajsberg algebra. Moreover, we have given some of the characterizations of fuzzy associative WI-ideal.

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