Change in Semigraph Energy Due to Edge Deletion and its Relation with Distance Energy

Hanumesha. A. G, Meenakshi. K

***Abstract*: If there is an adjacency matrix A, the sum total of the singular values of A is known as the graph energy. We can find the change in energy of a graph by removing the edges using the inequality theorem on singular values. In this paper we discuss about the change in semigraph energy due to deletion of edges and its relation with distance energy.**

***Index Terms*—Semigraph, Adjacency matrix, Energy of graph, Energy of Semigraph, Distance energy.**

I. INTRODUCTION

Semigraphs, the generalization of graphs was introduced by E.Sampathkumar. Using Semigraphs, we can connect many points by an edge and because of this fact, they have more uses than graphs.. In the second section, we study some preliminary definitions on semigraphs and singular value inequality for a matrix sum. In the third section, we study energy of graph, distance energy of semigraph, energy of a semigraph and then discuss the theorems on semigraph energy change due to the deletion of an edge and its relation with distance energy.

We have a look at the basics of semigraphs in the following section.

A Semigraph

An ordered pair (V, X) in which V is a nonempty set of elements called as vertices and the set of n tuples, known as edges is the set X, of distinct vertices, obeying the following conditions where the minimum of n is 2, is a semigraph.

i)If there are two edges then they have maximum one vertex in common ii) Two elements of the edge set are equal $(w_1,w_2,\ldots.w_n)$ and $(v_1,v_2,\ldots.v_m)$ are equal if a) m = n and b) either $w_i=v_i$ for i = 1 to n or $w_i=v_{n+1-i}$ for i = 1 to n. For the edge $E=(u_1,u_2,\ldots.u_n)$ w_1 and w_n are the end vertices and are the $w_2,\ldots.w_{n-1}$ are the middle end vertices of an edge.

B.Adjacent vertices

If there are two vertices in a semigraph and if they belong to the same edge then they are known as adjacent and also if they are consecutive then they are adjacent consecutively.

C.Cardinality

The number of elements of the vertex set lying on that edge is called as the cardinality of an edge.

D.Adjacent edges

If there is a vertex common in two edges, then the edges are said to be adjacent.

E.Edge Partial

An edge is said to be edge partial if it is a 1+k-j tuple $(v_{ij},v_{ij+1},\ldots v_{ik})$ where $1 \le j < k \le n$

F.Edge full

Any edge under consideration is said to be edge full of a semigraph.

G.Edge partial/Edge full

Any edge which is either an edge partial or an edge full is said to be edge partial or edge partial.

H.Regular semigraph

If all the vertices of a semigraph are of same degree, then it is said to be regular.

In this section, we discuss singular value inequality and some definitions on energy of graph and semigraphs and then discuss the theorems on change in energy of semigraph due to the deletion of an edge and its relation with energy distance.

A.Matrix sum inequality with respect to the singular values

Let the n by n complex matrix be X and let us denote its singular values by

$s_1(X) \ge s_2(X) \ge s_3(X) \ge \ldots\ldots\ldots \ge s_n(X) \ge 0$. Let there be only real eignevalues in X, that is $\lambda_1(X) \ge \lambda_2(X) \ge \ldots. \ge \lambda_n(X)$. Consider positive semidefinite $|X| = \sqrt{XX^*}$ where $\lambda_i(X) = s_i(X)$ for all i. Then the **Matrix sum inequality qwith respect to the singular values** is $\sum_{i=1}^{n} s_i(A+B) \le \sum_{i=1}^{n} s_i(A) + \sum_{i=1}^{n} s_i(B)$

B. Adjacency Matrix of a Semigraph

Consider a semigraph SG(V,X). Let $V = \{1,2,\ldots.p\}$ be vertex set and $X=\{e_1,e_2,\ldots e_q\}$, the edge set where $e_j = (i_{1,}i_2,\ldots\ldots i_j)$ and $i_{1,}i_2,\ldots\ldots i_j$ are distinct elements of V, then the $p \times p$ matrix A is the Adjacency matrix of semigraph $SG(V, X)$ whose entries are given by

$a_{ij} =$ cardinality of fp edge $(v_i, v_j) - 1$, if v_i and v_j are adjacent

= 0 , otherwise.

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Hanumesha.A.G, Assistant Professor, SJC Institute of Technology
Meenakshi.K, Associate professor, CMR Institute of Technology

C.Energy of a graph

Consider the graph G not directed, not infinite and not multiple graph with number of vertices n and number of edges m. Consider the adjacency matrix $A=\left(a_{ij}\right)$ of graph G , then the eigenvalues assumed in non-increasing order $\lambda_1,\lambda_2,.....\lambda_n$ of $A(G)$, are the graph eigenvalues. Then $E(G)$ graph energy G , is defined as $E(G)$ = $\sum_{i=1}^{n}|\lambda_i|$. The spectrum G is the set $\{\lambda_1,\lambda_2,....\lambda_n\}$ denoted by Spec G . If G has distinct eigenvalues say, $\lambda_1>\lambda_2>....>\lambda_n$ and if their multiplicities are $m(\lambda_i)$ then

$$Spec\,G=\begin{pmatrix}\lambda_1 & \lambda_2 & .. & .. & \lambda_n\\ m(\lambda_1) & m(\lambda_2) & .. & .. & m(\lambda_n)\end{pmatrix}$$

The spectrum of the graph does not depend on the labeling of the vertex set of the graph. Since we have the matrix symmetric and real with the trace zero, we have sum of the real real eignvalues to be zero.

D A kind of energy with respect to the distance in a semigraph

If SG is a simple connected semigraph and the vertices are labelled as $v_1, v_2, \cdots v_n$. then the matrix of a semigraph with respect to the distance, SG is given by a square matrix $D(G)=\left(d_{ij}\right)$ in which the entries are the distance between the vertices v_i and v_j in SG . The eigenvalues of the matrix we consider $\mu_1,\mu_2,\cdots\mu_n$ are said to be the **distance eigenvalues**. As we have a matrix which is symmetric, we have real eigenvalues in order $\mu_1\geq\mu_2\geq...\geq\mu_n$. The energy with respect to distance of a semigraph $E_D(SG)$ is defined as

$$E_D(SG)=\sum_{i=1}^{n}|\mu_i|.$$

E.Energy of a Semigraph

Let SG be not directed, not infinite and not a multiple semigraph with number of vertices n and number of edges m. Consider $A=\left(a_{ij}\right)$ the adjacency matrix of semigraph SG. The eigenvalues $\eta_1,\eta_2,.....\eta_n$ of $A(SG)$, taken not increasing order, are the eigenvalues of the semigraph SG. *Energy* of a semigraph SG, denoted by $E(SG)$ is defined as $E(SG)=\sum_{i=1}^{n}|\eta_i|$. The set $\eta_1,\eta_2,.....\eta_n$ is the spectrum of SG and is denoted by Spec SG . If the eigenvalues of SG are distinct say, $\eta_1>\eta_2>....>\eta_n$ with their multiples $m(\eta i)$ then we write

$$Spec\,SG=\begin{pmatrix}\eta_1 & \eta_2 & .. & .. & \eta_n\\ m(\eta_1) & m(\eta_2) & .. & .. & m(\eta_n)\end{pmatrix}$$

The spectrum of the above graph does not depend on the labeling of the vertex set of the graph. Since we have the matrix symmetric and real with the trace zero, we have sum of the real real eignvalues to be zero.

F.Theorem 3.4

If SH is a non-empty induced subsemigraph of a simple connected regular semigraph SG , then $E(SG)-E(SH)\leq E(SG')\leq E(SG)+E(SH)$

Proof

SG is a connected simple semigraph. SH be an induced subsemigraph of SG ,containing all edges of SG connecting two vertices of SH Let $SG-SH$ be the semigraph, having got from SG removing all vertices of SH and the edges incident with SH . If there are two semigraphs $SG1$ and $SG2$ semigraphs with out any vertices in common and if we consider $SG1\oplus SG2$ as the semigraph with vertex set and the edge set $V(SG1)\cup V(SG2)$; $E(SG1)\cup E(SG2)$ respectively. Hence

$$A(SG1\oplus SG2)=A(SG1)\oplus A(SG2)$$

$$A(SG)=\begin{pmatrix}A(SH) & X^T\\ X & A(SG-SH)\end{pmatrix}$$

$$=\begin{pmatrix}A(SH) & 0\\ 0 & 0\end{pmatrix}+\begin{pmatrix}0 & X^T\\ X & 0\end{pmatrix}$$

In which the edges joining SH and $SG-SH$ *is* X.

Also if $A(SG')=\begin{pmatrix}0 & X^T\\ X & 0\end{pmatrix}$

Using the inequality theorem of matrices with respect to singular value

$E(SG)\leq E(SH)+E(SG')$, which gives one part of the inequality

$E(SG)-E(SH)\leq E(SG')$

$$A(SG')=A(SG)+\begin{pmatrix}-A(SH) & 0\\ 0 & 0\end{pmatrix}$$

By the inequality theorem with respect to singular values,

$E(SG')\leq E(SG)+E(SH)$

$E(SG')\leq E(SG)+E(SH)---ii$

From i and ii , it follows

$E(SG)-E(SH)\leq E(SG')\leq E(SG)+E(SH)$

Both the left and right equality holds when $E(SH)=\phi$

G.Theorem 3.5 If SH is a non-empty induced subsemigraph of a simple connected semigraph SG. Then

$$E_D(SG) - E_D(SH) \le E(SG') < E_D(SG) + E_D(SH)$$

. **Proof**

SG is a connected simple semigraph. SH be an induced subsemigraph of SG,containing all edges of SG joining two vertices of SH Let $SG - SH$ denote the semigraph, having got from SG removing all vertices of SH and the edges that are incident with SH. If $SG1$ and $SG2$ are two semigraphs not having any vertices in common, and if we consider $SG1 \oplus SG2$ the semigraph with vertex set and the edge set $V(SG1) \cup V(SG2)$; $E(SG1) \cup E(SG2)$ respectively. Hence

$$A(SG1 \oplus SG2) = A(SG1) \oplus A(SG2)$$

$$D(SG1 \oplus SG2) = D(SG1) \oplus D(SG2)$$

$$D(SG) = \begin{pmatrix} D(SH) & X^T \\ X & D(SG - SH) \end{pmatrix}$$

$$= \begin{pmatrix} D(SH) & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & X^T \\ X & D(SG - SH) \end{pmatrix}$$

X represents edges connecting SH and $SG - SH$.

$$D(SG) = \begin{pmatrix} D(SH) & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & X^T \\ X & A(SG - SH) \end{pmatrix}$$

Also if $A(SG') = \begin{pmatrix} 0 & X^T \\ X & A(SG - SH) \end{pmatrix}$

$$D(SG) = \begin{pmatrix} D(SH) & 0 \\ 0 & 0 \end{pmatrix} + A(SG')$$

By singular value inequality theorem

$$E_D(SG) \le E_D(SH) + E(SG')$$

$$E_D(SG) - E_D(SH) \le E(SG') --- i$$

$$A(SG') = D(SG) + \begin{pmatrix} -D(SH) & 0 \\ 0 & 0 \end{pmatrix}$$

By singular value inequality theorem,

$$E(SG') \le E_D(SG) + E_D(SH)$$

$$E(SG') \le E_D(SG) + E_D(SH) --- ii$$

From i and ii ,wehave

$$E_D(SG) - E_D(SH) \le E(SG') \le E_D(SG) + E_D(SH)$$

Both the left and right equality holds when $E_D(SH) = \phi$

II. CONCLUSION

We find that energy of semigraphs increases or decreases or remains the same due to edge deletion. We can study the semigraph energy change due to vertex deletion using singular value inequality. We can study its relation with other forms of energies of semigraphs due to edge deletion and vertex deletion

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