FS-Cartesian Product Topological Space and its Compactness

Vaddiparthi Yogeswara, K.V. Umakameswari, D. Raghu Ram, Ch. Ramasanyasi Rao , K. Aruna kumari

Abstract: ***For any nonempty family*** $\{(\mathcal{B}_i, \mathfrak{T}_i)\}$ ***of compact FsB-Toplogical Spaces, the corresponding Fs-product space is also compact.***

Index Terms: ***Fs-Set, Fs-Subset, (b,β) object, Fs-Point, FsB-Toplogical Space.***

I. INTRODUCTION

Axiom choice is not true in the theory of L-Fuzzy sets .Nistla V.E.S Murthy [10] proved Axiom Choice of fuzzy sets in his theory of F-sets. VaddiparthiYogeswara[2] etc … developed the theory of Fs-sets with the goal of introducing the complement of a fuzzy set which was not satisfactorily explained by previous relevant theories .Also VaddiparthiYogeswara, BiswajitRath ,Ch.RamaSanyaasiRao ,K.V.UmaKameswari,D.Raghu Ram introduced the concept of FsB-topological Space on a given Fs –subset of an Fs-set and also they introduced FsB-subspace in the same paper .Fs-points and Fs-point set FSP($\mathcal{W}$) are introduced by VaddiparthiYogeswara etc…[2] and based on Fs-set theory they defined a pair of relations between P(FSP($\mathcal{W}$))and $\mathcal{L}(\mathcal{W})$. Here FSP($\mathcal{W}$) stands for Fs-Point set of $\mathcal{W}$, $\mathcal{L}(\mathcal{W})$ stands for collection of allFs-subsets of $\mathcal{W}$ and P(FSP($\mathcal{W}$)) is power set of FSP($\mathcal{W}$) and proved one of them is a '∧'- complete homomorphism and other is '∨'- complete homomorphism and searched some properties of these relations between complemented constructed crisp sets and Fs-complemented sets through thesehomomorphismand ultimately they proved a representation theorem connecting Fs-subsets of $\mathcal{W}$ to crisp subsets of FSP($\mathcal{W}$) via homomorphisms.For a given non-empty family of compact Fs-toplogicalspaces , we prove in this paper their Fs-Cartesian Product space is also compact. Fs-Sets, Fs- Set functions etc… in brief are explained in first four sections of this paper. '∪' and '⋂' sands for natural set union and Fs-union and Similarly '⋂'. M_A or 1_A sands for largest element of a given complete Boolean Algebra L_A. For all lattice theoretic and relevant Properties one can refer [5],[8],[15],[16],[17].𝕊ET, the category of sets with usual maps between crisp sets.ℂBOO, the category of complete Boolean algebras with complete homomorphism between complete Boolean algebras.$(\prod_{i\in I} A_i, (P_i)_{i\in I})$is the product of $(A_i)_{i\in I}$ in 𝕊ET. Meanings of all the following things can known from [2]. (i) 𝕊ET (ii) ℂBOO (iii) Fs-Cartesian Product (iv) Axiom choice.

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Vaddiparthi Yogeswara, Department of Mathematics, GIT, GITAM Deemed to be University, Visakhapatnam-530045, Andhra Pradesh, India
K.V. Umakameswari Research Scholar, Dept. of Applied Mathematics, GIS, GITAM Deemed to be University, Visakhapatnam 530045, A.P, India
D. Raghu Ram, Research Scholar: Dept. of Applied Mathematics, GIS, GITAM Deemed to be University, Visakhapatnam 530045, A.P, India
Ch. Ramasanyasi Rao, Dept. of Applied Mathematics, MVR DEGREE&P.G College,Gajuwaka,Visakhapatnam-530026,A.P, India
K. Aruna kumari, Dept. of Mathematics, GIT, GITAM University Visakhapatnam 530045, A.P ,

SECTION-1

1.1 Fs-set: : A four tuple of the form $\mathcal{W} = (W_1, W, \overline{W}(\mu_{1W_1}, \mu_{2W}), L_W)$ is an Fs-set iff, $W \sqsubseteq W_1 \sqsubseteq U$

(1) L_W is a complete Boolean Algebra

(2) $\mu_{1W_1}: W_1 \to L_W$,$\mu_{2W}: W \to L_W$ are mappings such that

$$\mu_{1W_1}|W \geq \mu_{2W}$$

(3) $\overline{W}$: W→ L_W is defined by

$$\overline{W}x = \mu_{1W_1}x \wedge (\mu_{2W}x)^c \text{ for each } x \in W$$

Where W is a non-void subset of some universal set U

1.2 Fs-subset: Suppose $\mathcal{W} = (W_1, W, \overline{W}(\mu_{1W_1}, \mu_{2W}), L_W)$ and $\mathcal{U} = (U_1, U, \overline{U}(\mu_{1U_1}, \mu_{2U}), L_U)$ are two Fs-sets. We say $\mathcal{U}$ is anFs-subset of $\mathcal{W}$, in symbol, we write $\mathcal{U} \sqsubseteq \mathcal{W}$, iff

(1) $U_1 \sqsubseteq W_1$, $W \sqsubseteq U$

(2) L_U is a complete subalgebra of L_W or $L_U \leq L_W$

(3) $\mu_{1U_1} \leq \mu_{1W_1}|U_1$, and $\mu_{2U}|W \geq \mu_{2W}$

1.3 ArbitraryFs-unions and arbitrary Fs-intersections

For any $(\mathcal{U}_i)_{i\in I}, \mathcal{U}_i = (U_{1i}, U_i, \overline{U}_i(\mu_{1U_{1i}}, \mu_{2U_i}), L_{U_i}) \sqsubseteq \mathcal{W} = (W_1, W, \overline{W}(\mu_{1W_1}, \mu_{2W}), L_W)$, $i \in I$

(1): $\bigsqcup_{i\in I} \mathcal{U}_i = \varphi_{\mathcal{W}}$, for $I = \varphi$

(2): If $I \neq \varphi$, $\bigsqcup_{i\in I} \mathcal{U}_i = \mathcal{U} = (U_1, U, \overline{U}(\mu_{1U_1}, \mu_{2U}), L_U)$, where

(a) $U_1 = \bigsqcup_{i\in I} U_{1i}$, $U = \bigsqcap_{i\in I} U_i$

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(b) $L_U = \curlyvee_{i\in I} L_{U_i}$ = complete subalgebra generated by $\curlyvee_{i\in I} L_i (L_i = L_{U_i})$

(c) $\mu_{1U_1}: U_1 \to L_U$ is defined by

$$\mu_{1U_1}\mathcal{x} = \left(\curlyvee_{i\in I} \mu_{1U_{1i}}\right)\mathcal{x}$$
$$= \curlyvee_{i\in I_{\mathcal{x}}} \mu_{1U_{1i}}\mathcal{x}\text{, where } I_{\mathcal{x}} = \{i \in I \mid \mathcal{x} \in U_i\}$$

$\mu_{2U}: U \to L_U$ is defined by

$$\mu_{2U}\mathcal{x} = \left(\curlywedge_{i\in I} \mu_{2U_i}\right)\mathcal{x}$$
$$= \curlywedge_{i\in I} \mu_{2U_i}\mathcal{x}$$

$\overline{U}: U \to L_U$ is defined by

$$\overline{U}\mathcal{x} = \mu_{1U_1}\mathcal{x} \curlywedge (\mu_{2U}\,\mathcal{x})^c$$

1.19.2 Definition

(1) : $\sqcap_{i\in I} \mathcal{U}_i = \mathcal{W}$, for $I = \varphi$

(2) : Suppose

$$\sqcap_{i\in I} U_{1i} \sqsupseteq \sqcup_{i\in I} U_i, \curlywedge_{i\in I} \mu_{1U_{1i}} \Big| \left(\sqcup_{i\in I} U_i\right) \geq \curlyvee_{i\in I} \mu_{2U_i}$$

$\sqcap_{i\in I} \mathcal{U}_i$ as

$$\sqcap_{i\in I} \mathcal{U}_i = \mathcal{V} = (V_1, V, \overline{V}(\mu_{1V_1}, \mu_{2V}), L_V)$$

(a') $V_1 = \sqcap_{i\in I} U_{1i}, V = \sqcup_{i\in I} U_i$

(b') $L_V = \curlywedge_{i\in I} L_{U_i}$

(c') $\mu_{1V_1}: V_1 \to L_V$ is defined by

$$\mu_{1V_1}\mathcal{x} = \left(\curlywedge_{i\in I} \mu_{1U_{1i}}\right)\mathcal{x}$$
$$= \curlywedge_{i\in I} \mu_{1U_{1i}}\mathcal{x}$$

$\mu_{2V}: V \to L_V$ is defined by

$$\mu_{2V}\mathcal{x} = \left(\curlyvee_{i\in I} \mu_{2U_i}\right)\mathcal{x}$$
$$= \curlyvee_{i\in I_{\mathcal{x}}} \mu_{2U_i}\mathcal{x}\text{, where } I_{\mathcal{x}} = \{i \in I \mid \mathcal{x} \in U_i\}$$

$\overline{V}: V \to L_V$ is defined by

$$\overline{V}\mathcal{x} = \mu_{1V_1}\mathcal{x} \curlywedge (\mu_{2V}\,\mathcal{x})^c$$

(3) $\sqcap_{i\in I} U_{1i} \not\sqsupseteq \sqcup_{i\in I} U_i$ or $\curlywedge_{i\in I} \mu_{1U_{1i}} \Big| \left(\sqcup_{i\in I} U_i\right) \not\geq \curlyvee_{i\in I} \mu_{2U_i}$

Define

$$\sqcap_{i\in I} \mathcal{U}_i = \varphi_{\mathcal{W}}$$

Agree

$\sqcap_{i\in I} \mathcal{U}_i = \varphi_1$ = Type-I Void set if $\sqcap_{i\in I} \mathcal{U}_i = \Omega_\varphi$

Fs-complement of an Fs-subset

1.6 Definition

Consider a particular Fs-set $\mathcal{W} = (W_1, W, \overline{W}(\mu_{1W_1}, \mu_{2W}), L_W)$, $W \neq \Phi$, where

(i) $W \subseteq W_1$

(ii) $L_W = [0, M_A]$, M_A is the largest element of L_A

(iii) $\mu_{1W_1} = M_A$, $\mu_{2W} = 0$

$\overline{W}x = \mu_{1W_1}x \wedge (\mu_{2W}x)^c = M_A$ for each $x \in A$

Given $\mathcal{V} = (V_1, V, \overline{V}(\mu_{1V_1}, \mu_{2V}), L_V)$. We define Fs-complement of $\mathscr{B}$ in $\mathcal{A}$, denoted by $\mathcal{V}^{C_{\mathcal{A}}}$ for V=W and $L_V = L_W$ as

$\mathcal{V}^{C_{\mathcal{A}}} = \mathcal{U} = (U_1, U, \overline{U}(\mu_{1U_1}, \mu_{2U}), L_U)$, where

(a') $U_1 = C_A V_1 = V_1^c \cup W, U = V = W$ where $V_1^c = W_1 - V_1$

(b') $L_U = L_W$

(c') $\mu_{1U_1}: U_1 \to L_W$ is defined by

$$\mu_{1U_1}x = M_A$$

$\mu_{2U}: W \to L_W$ is defined by

$$\mu_{2U}x = \overline{V}x = \mu_{1V_1}x \wedge (\mu_{2V}x)^c$$

$\overline{U}: W \to L_W$ is defined by

$$\overline{U}x = \mu_{1U_1}x \wedge (\mu_{2U}x)^c = M_A \wedge (\overline{V}x)^c = (\overline{V}x)^c.$$

1.7 Fs-empty set: For some $L_\Omega, L_\Omega \leq L_W$, $\Omega_\varphi = (\Omega_1, \Omega, \overline{\Omega}(\mu_{1\Omega_1}, \mu_{2\Omega}), L_\Omega)$ with conditions

(a) $\Omega \nsubseteq \Omega_1$ or Ω is a void set

(b) $\mu_{1\Omega_1}\mathcal{x} \ngeq \mu_{2\Omega}\mathcal{x}$, for some $\mathcal{x} \in \Omega \sqcap \Omega_1$ or $\mu_{2\Omega}$ is a void function.

And throughout this thesis, this specific $\mathcal{X}$ is denoted by φ_1 and we agree that

$$\varphi_1 \sqsubseteq \mathcal{U}, \text{for any Fs} - \text{subset U}$$

1.7 **Definition**: If $\mathcal{Y} = (Y_1, Y, \overline{Y}(\mu_{1Y_1}, \mu_{2Y}), L_Y)$ is an Fs-subset of $\mathcal{U}$, with the following properties

(a') $\mathcal{U} \sqsubseteq \mathcal{W}$

(b') $Y_1 = Y = W$

(c') $L_Y \leq L_W$

(d') $\overline{Y} = 0$ or $\mu_{1Y_1} = \mu_{2Y}$

Then, we say that $\mathcal{Y}$ is an Type-II Void set and is denoted by φ_2

SECTION-2

(b, β)- Object

2.1Definition Let $b \in A, \beta \in L_A$ such that $\beta \leq \overline{A}b$. we define a (b, β)-object, denoted by (b, β) itself as follows

for $A \subseteq B \subseteq B_1 \subseteq A_1, L_B \leq L_A$, such that $\mu_{1B_1}x, \mu_{2B}x \in L_B(b, \beta) = (B_1, B, \overline{B}(\mu_{1B_1}, \mu_{2B}), L_B)$

$$\mu_{1B_1}x = \begin{cases} \mu_{2A}x, & x \neq b, x \in A \\ \beta \vee \mu_{2A}b, & x = b \\ \alpha, & x \notin A, x \in A_1 \end{cases}$$ And $$\mu_{2B}x = \begin{cases} \mu_{2A}x, & x \in A \\ \alpha, & x \notin A, x \in B \end{cases}$$

Here $\alpha \in L_A$ is fixed and $\alpha \leq \mu_{1A_1}x, \forall x \in A_1$

2.2 R(b, β)Relation : For any (b, β) objects $\mathcal{B}_1 = (B_{11}, B_1, \overline{B}_1(\mu_{1B_{11}}, \mu_{2B_1}), L_{B_1})$ and $\mathcal{B}_2 = (B_{12}, B_2, \overline{B}_2(\mu_{1B_{12}}, \mu_{2B_2}), L_{B_2})$ of $\mathcal{A}$, we say that $\mathcal{B}_1 R(b, \beta) \mathcal{B}_2$ if, and only if

$\mu_{1B_{11}}x = \mu_{2B_1}x, x \neq b$ and $\forall x \in B_1$ and $\mu_{1B_{12}}x = \mu_{2B_2}x, x \neq b$ and $\forall x \in B_2$ and

$\mu_{1B_{11}}b = \mu_{1B_{12}}b = \beta \vee \mu_{2A}b$ and $\mu_{2B_1}b = \mu_{2B_2}b = \mu_{2A}b$.
We can easily show that $R(b, \beta)$ is an equivalence relation

2.3 Fs-point : The equivalence class corresponding to (b, β) is denoted by χ_b^{β} or (b, β).We define this χ_b^{β} is an Fs-point of $\mathcal{A}$.Set of all Fs-point of $\mathcal{A}$ is denoted by FSP($\mathcal{A}$).

2.4 Definition For any $\mathcal{V} \subseteq \mathcal{W}$

$$\text{Define } \mathcal{V}^{\sim} = \begin{cases} \Phi & \text{if } \mathcal{V} = \Phi_{\mathcal{A}} \\ \left\{ \chi_b^{\beta} | b \in V, \beta \in L_V, \beta \le \overline{V}b \right\} & \text{otherwise} \end{cases}$$

where $\mathcal{V} = (V_1, V, \overline{V}(\mu_{1V_1}, \mu_{2V}), L_{\mathcal{V}})$
Since for any $a \in W, \chi_b^{\beta} \subseteq \mathcal{V}$ if $\mathcal{V} \subseteq \mathcal{W}$ exists.
Hence $\chi_b^0 \in \mathcal{V}^{\sim}$ for any $\mathcal{V} \subseteq \mathcal{W}$ if $\mathcal{V}$ exists. We call χ_b^0 as trivial Fs-point

SECTION-3

3.1 FsB-Toplogical Space : Suppose $\mu_{1A_1} = 1, \mu_{2A} = 0$ in $\mathcal{A}$. $\mathfrak{T} \subseteq \mathcal{L}(\mathcal{W})$ is said to be FsB-toplogy if, and only if

1) $(\mathcal{B}_i)_{i \in I} \subseteq \mathfrak{T} \Rightarrow \bigcup_{i \in I} \mathcal{B}_i \in \mathfrak{T}$
2) $(\mathcal{B}_i)_{i \in I}$, I is finite set $\Rightarrow \bigcap_{i \in I} \mathcal{B}_i \in \mathfrak{T}$.

The pair $(\mathcal{A}, \mathcal{T})$ is called an FsB-topological space.
Elements of $\mathfrak{T}$ are called FsB-open ses or FsB-open subset of $\mathcal{A}$.

3.2 FsB- Product topological Space : $\mathcal{S} = \prod_{i \in I} \mathcal{G}_i$ with $\mathcal{G}_i$ is open $\mathcal{A}_i$.
Every component of RHS is $\mathcal{A}_i$ for each $j \neq I$ and at the j^{th} place $\mathcal{G}_i$ is there.
$\mathfrak{G} = \{ \mathcal{S} \}$ is called defining FsB-open subbase for FsB-product topology on $\prod_{i \in I} \mathcal{A}_i$.
The FsB-open base $\mathfrak{B} = \{ \prod_{i \in I} \mathcal{G}_i \, | \mathcal{G}_i = \mathcal{A}_i$ for all $i \in I - \{i_1, i_2, i_3 \dots i_n\}, i_1, i_2, i_3, \dots i_n \in I \}$ is called defining FsB-open base for the FsB-topology generated by $\mathfrak{B}$.
The $\mathfrak{B} = \{\prod_{i \in I} \mathcal{F}_i \, | \mathcal{F}_i = \mathcal{A}_i$ for all $i \neq i_0$, $\mathfrak{s}_{i_0}$ is closed in $\mathcal{B}_{i_o}$ $\}$ is called defining FsB-closed sets for the Product topology.
The FsB-topology on $\prod_{i \in I} \mathcal{A}_i$ generated by $\mathfrak{B}$ is called FsB-product topology
Let $\mathcal{A}_i = (A_{1i}, A_i, \overline{A}_i(\mu_{1A_{1i}}, \mu_{2A_i}), L_{A_i})$ be a family of FsB-topological spaces.
Let $\prod_{i \in I} \mathcal{A}_i$ be Fs-Cartesian Product of the family $\{\mathcal{A}_i \}_{i \in I}$.
Let $\mathfrak{G} = \{ \mathcal{S} \}$ where $\mathcal{S} = \prod_{i \in I} \mathcal{B}_i$ where $\mathcal{B}_i = \begin{cases} \mathcal{A}_i & i \neq i_1 \\ \mathcal{G}_i & i = i_1 \end{cases}$ and $\mathcal{G}_{i_1}$ be FsB-open in $\mathcal{A}_{i_1}$, $i_1 \in I$
Where $\mathcal{G}_{i_1} = \left(G_{1i_1}, G_{i_1}, \bar{\mathcal{G}}_{i_1}\left(\mu_{1G_{1i_1}}, \mu_{2G_{i_1}}\right), L_{G_{i_1}}\right)$ $\mathfrak{B} = \{\mathcal{S}_1 \cap \mathcal{S}_2 \cap \mathcal{S}_3 \cap \dots \mathcal{S}_n | \mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3 \dots . \mathcal{S}_n\}$

3.3 Theorem : $\mathfrak{B}$ is an FsB-open base for FsB- product topology on $\mathcal{W}$.

3.4 Theorem : AnyFsB-topological space $(\mathcal{B}, \mathfrak{T})$ is compact if and only if every non empty family of defining FsB-sub basic closed sets with finite intersection property has nonempty intersection.

3.5 Theorem :$(\mathcal{B}, \mathfrak{T})$ is compact if and only if every non empty family of FsB-sub basic closed sets with finite intersection property has nonempty intersection.
Proof : Sufficient to Prove that everynon-empty family of definingFsB- sub basic closed sets with finite intersection property has non-empty intersection.
Consider $\{\mathcal{F}_j\}$, a non- empty family of non-emptydefiningFsB-sub basic closed sets in $(\mathcal{B}, \mathfrak{T})$.
Then for each $j \in J$, $\mathcal{F}_j = \prod_{i \in I} \mathcal{F}_{ji}$ where $\mathcal{F}_{ji} = \begin{cases} \mathcal{B}_i, & \text{for all } i \in I \; i \neq i_o \\ \mathcal{F}_{ji_0}, & \text{a sub basic closed set } in \; \mathcal{B}_{i_o} \end{cases}$

Then , for each i^{th}Fs-projection $\prod_i : \mathcal{B} = \prod_{i \in I} \mathcal{B}_i \longrightarrow \mathcal{B}_i$

$\prod_i (\mathcal{F}_j) = \mathcal{F}_{ji}$ is non emptyFsB-sub basic closed set in $\mathcal{B}_i$.

In Particular, $\prod_{i_0} (\mathcal{F}_j) = \mathcal{F}_{ji_0}$ is non-empty FsB-sub basic closed set in $\mathcal{B}_{i_o}$.

Hence $\{\mathcal{F}_{ji_0}\}_{j \in J} = \mathfrak{F}_{i_0}$ is a nonempty family of nonempty FsB-closed sets in $\mathcal{B}_{i_o}$.

Also,every finite subfamily of $\mathfrak{F}_{i_0}$ has nonempty intersection.
Since $\mathcal{B}_{i_o}$ is compact, we have $\bigcap \mathfrak{F}_{i_0} = \bigcap_{j \in J} \mathcal{F}_{ji_0}$ is non empty.
Fix $\chi_{\alpha i_0}^{\alpha i_0}$ in $\mathcal{B}_{i_o}{}^{\sim}$. Then $(\chi_{\alpha i_0}^{\alpha i_0})_{i \in I} \in (\prod_{i \in I} \mathcal{B}_i)^{\sim} = \prod_{i \in I} \mathcal{B}_i{}^{\sim}$ (3.4)
Hence $(\chi_{\alpha i_0}^{\alpha i_0})_{i \in I} \in (\bigcap_{j \in J} \mathcal{F}_j)^{\sim}$. So, that $\bigcap_{j \in J} \mathcal{F}_j$ is non-emptyin $\prod_{i \in I} \mathcal{B}_i$.
Hence $\prod_{i \in I} \mathcal{B}_i$ is compact.

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