Einstein Operations of Intuitionistic Fuzzy Matrices

T.M. Selvarajan, S. Sriram, R.S. Ramya

Abstract--- In this paper, the authors defined the Einstein operations of fuzzy matrices, intuitionistic fuzzy matrices and proved several properties of them, particularly those involving the intuitionistic fuzzy implication with other operations.

Keywords:Fuzzy Matrix, Intuitionistic Fuzzy Matrix, Einstein Sum and Einstein Product.

I. INTRODUCTION

It is well known that matrices play major role in various areas such as Mathematics, Physics, Statistics, Engineering, Social sciences and many others. However, we cannot successfully use classical matrices because of various types of uncertainties present in real world situations. Now a days probability, Fuzzy sets, Intuitionistic fuzzy sets, Vague sets are used as Mathematical tools for dealing uncertainties. Fuzzy matrices arise in many applications, one of which is as adjacency matrices of fuzzy relations. A fuzzy matrix is a matrix over the fuzzy algebra $\mathfrak{I}$=[0,1] under the fuzzy operations formulated by Zadeh in 1965 [8]. Several authors presented a number of results on fuzzy matrices.In 1977, Thomoson [6] studied the behaviour of powers of fuzzy matrices using max-min composition. The theory of fuzzy matrices was systematically developed by Kim and Roush in [2] . Ragab and Emam [4] studied some properties of the min-max compositions of fuzzy matrices; it can be regarded as the dual of max-min composition of fuzzy matrices .Among the well known operations which can be performed on fuzzy matrices are the operations $\vee$, $\wedge$and complementation. In addition to these operations, the operations $\oplus$and $\odot$are introduced by Shyamaland Pal [6]. Also several properties on $\oplus$and $\odot$, some results on existing operators along with these operations are studied. Wang and Liu[8] introduced some Einstein operations of intuitionistic fuzzy sets and analyze some desirable properties of the proposed operations. In [5] Selvarajan et.al studied Einstein operations to fuzzy matricesand proved several properties of them. In this paper, we extend the Einstein operations to intuitionistic fuzzy matricesand proved several properties of them.

II. FUZZY MATRICES

Definition:If $A$and B are two fuzzy matrices of same size, where $A=[a_{ij}]$ and $B=[b_{ij}]$ then

(i) The Einstein sum of A and B is defined by $A\oplus_\varepsilon B=\left[\frac{a_{ij}+b_{ij}}{1+a_{ij}b_{ij}}\right]$

(ii) The Einstein product of A and B is defined by

$$A\otimes_\varepsilon B=\left[\frac{a_{ij}b_{ij}}{1+(1-a_{ij})(1-b_{ij})}\right].$$

Definition:If $A=[a_{ij},a'_{ij}]$and $B=[b_{ij},b'_{ij}]$ are two intuitionistic fuzzy matrices (IFMs) of same size, then

(i) The Einstein sum of A and B is defined by

$$A\oplus_\varepsilon B=\left[\frac{a_{ij}+b_{ij}}{1+a_{ij}b_{ij}},\frac{a'_{ij}b'_{ij}}{1+(1-a'_{ij})(1-b'_{ij})}\right]$$

(ii) The Einstein product of A and B is defined by

$$A\otimes_\varepsilon B=\left[\frac{a_{ij}b_{ij}}{1+(1-a_{ij})(1-b_{ij})},\frac{a'_{ij}+b'_{ij}}{1+a'_{ij}b'_{ij}}\right]$$

III. NECESSITY AND POSSIBILITY OPERATORS

The necessity and possibility operators for an IFM are defined by Pal [14]. In this section, the algebraic properties of necessity and possibility operators of IFMs with respect to Einstein operations are discussed.

Definition

Let A be an IFM of size $m\times n$. Then the necessity operator $\square$ of A is defined by $\square A=(\langle a_{ij},1-a_{ij}\rangle)$,the possibility operator $\lozenge$ of A is defined by $\lozenge A=(\langle 1-$ *aij', aij'.*

Theorem 1: For any two IFMs $A$and B of same size, the following results are hold hood:

$(i)\square(A\oplus_\varepsilon B)=\square A\oplus_\varepsilon\square B$

$(ii)\ \lozenge(A\oplus_\varepsilon B)=\lozenge A\oplus_\varepsilon\lozenge B.$

Proof

$$(i)\square(A\oplus_\varepsilon B)=\left(\langle\frac{a_{ij}+b_{ij}}{1+a_{ij}b_{ij}},1-\left(\frac{a_{ij}+b_{ij}}{1+a_{ij}b_{ij}}\right)\rangle\right)$$

$$=\left(\langle\frac{a_{ij}+b_{ij}}{1+a_{ij}b_{ij}},\frac{(1-a_{ij})(1-b_{ij})}{1+a_{ij}b_{ij}}\rangle\right)$$

$=\square A\oplus_\varepsilon\square B$

(ii) It can be proved analogously.

Analogously we can prove the following theorem.

Theorem 2: For any two IFMs $A$and B of same size, the following results are hold hood:

$(i)\square(A\otimes_\varepsilon B)=\square A\otimes_\varepsilon\square B$

$(ii)\ \lozenge(A\otimes_\varepsilon B)=\lozenge A\otimes_\varepsilon\lozenge B.$

Theorem 3: For any two IFMs $A$and B of same size, the following results are hold hood:

$$(i)\left(\square(A^C\oplus_\varepsilon B^C)\right)^C=\lozenge A\otimes_\varepsilon\lozenge B$$

$$(ii)\left(\square(A^C\otimes_\varepsilon B^C)\right)^C=\lozenge A\oplus_\varepsilon\lozenge B$$

Proof

$$(i)\square(A^C\oplus_\varepsilon B^C)=\left(\langle\frac{a'_{ij}+b'_{ij}}{1+a'_{ij}b'_{ij}},1-\left(\frac{a'_{ij}+b'_{ij}}{1+a'_{ij}b'_{ij}}\right)\rangle\right)$$

Manuscript received May15, 2019.

T.M. Selvarajan, Department of Mathematics, Noorul Islam Centre for Higher Education, Kanyakumari, India.

S. Sriram, Department of Mathematics, Annamalai University, Chidambaram, India.

R.S. Ramya, Department of Mathematics, Noorul Islam Centre for Higher Education, Kanyakumari, India.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

$$= \left(\langle\frac{a'_{ij}+b'_{ij}}{1+a'_{ij}b'_{ij}},\frac{(1-a'_{ij})(1-b'_{ij})}{1+a'_{ij}b'_{ij}}\rangle\right)$$

$$\left(\Box(A^C \oplus_\varepsilon B^C)\right)^C = \left(\langle\frac{(1-a'_{ij})(1-b'_{ij})}{1+a'_{ij}b'_{ij}},\frac{a'_{ij}+b'_{ij}}{1+a'_{ij}b'_{ij}}\rangle\right)$$
$$= \Diamond A \otimes_\varepsilon \Diamond B$$

(ii) It can be proved analogously.

Analogously we can prove the following theorem.

Theorem 4: For any two IFMs $A$and B of same size, the following results are hold hood:

$$(i)\left(\Diamond (A^C \oplus_\varepsilon B^C)\right)^C = \Box A \otimes_\varepsilon \Box B$$

$$(ii)\left(\Diamond (A^C \otimes_\varepsilon B^C)\right)^C = \Box A \oplus_\varepsilon \Box B$$

Define $nA$and A^n for any positive integer $n(n > 0)$ with respect to the Einstein operations, by using these definitions we have the following theorems.

Definition:For an IFM $A$and for any positive integer$n(n > 0)$

$$nA = \left(\langle\frac{(1+a_{ij})^n-(1-a_{ij})^n}{(1+a_{ij})^n+(1-a_{ij})^n},\frac{2(a'_{ij})^n}{(2-a'_{ij})^n+(a'_{ij})^n}\rangle\right)\text{and}$$

$$A^n = \left(\langle\frac{2(a_{ij})^n}{(2-a_{ij})^n+(a_{ij})^n},\frac{(1+a'_{ij})^n-(1-a'_{ij})^n}{(1+a'_{ij})^n+(1-a'_{ij})^n}\rangle\right)$$

Theorem 5: For an IFM $A$and for any positive integer $n(n > 0)$

$$(i)\Box(nA) = n\Box A$$
$$(ii)\ \Diamond(nA) = n\Diamond A.$$

Proof

$$(i)\Box(nA) = \left(\langle\frac{(1+a_{ij})^n-(1-a_{ij})^n}{(1+a_{ij})^n+(1-a_{ij})^n},1-\frac{(1+a_{ij})^n-(1-a_{ij})^n}{(1+a_{ij})^n+(1-a_{ij})^n}\rangle\right)$$
$$= \left(\langle\frac{(1+a_{ij})^n-(1-a_{ij})^n}{(1+a_{ij})^n+(1-a_{ij})^n},\frac{2(1-a_{ij})^n}{(1+a_{ij})^n+(1-a_{ij})^n}\rangle\right)$$
$$= n\Box A.$$

(ii) It can be proved analogously.

Theorem 6: For an IFM $A$and for any positive integer $n(n > 0)$

$$(i)\Box(A^n) = (\Box A)^n$$
$$(ii)\ \Diamond(A^n) = (\Diamond A)^n.$$

Proof: Similar to Theorem 5.

Theorem 7: For an IFM $A$and for any two positive integers $m > 0, n > 0$, if$m > n$ then $mA > nA$.

Proof: Let $A = (\langle a_{ij}, a'_{ij}\rangle)$ be an intuitionistic fuzzy matrix.Let$mA = (\langle c_{ij}, c'_{ij}\rangle), nA = (\langle d_{ij}, d'_{ij}\rangle)$. Let $f(x) = \frac{(1+a)^x-(1-a)^x}{(1+a)^x+(1-a)^x}$, $g(x) = \frac{2b^y}{(2-b)^y+b^y}$then we have

$$f'(x) = \frac{2(1+a)^x(1-a)^x\left(ln(1+a)-ln(1-a)\right)}{[(1+a)^x+(1-a)^x]^2}$$
$$> 0, a \in [0,1], x > 0,$$

$$g'(y) = \frac{2b^y(2-b)^y ln\frac{y}{2-y}}{[(2-b)^y+b^y]^2} < 0, b \in [0,1], y > 0.$$

Which show that $f(x)$ and $g(x)$ are increasing and decreasing functions respectively. So if $m > n$ then $f(m) > f(n)$ and $g(m) < g(n), i.e., c_{ij} > d_{ij}$ and $c'_{ij} < d'_{ij}$, thus $mA > nA$.

Theorem 8: For an IFM $A$and for any two positive integers$m > 0, n > 0$, if$m > n$ then $A^m > A^n$.

Proof: Similar to **Theorem 7**

Definition: The concentration of an IFM A is denoted by $CON(A)$ and is defined by$CON(A) = (\langle b_{ij}, b'_{ij}\rangle)$,where $b_{ij} = \frac{2(a_{ij})^2}{(2-a_{ij})^2+(a_{ij})^2}$, $b'_{ij} = \frac{(1+b'_{ij})^2-(1-b'_{ij})^2}{(1+b'_{ij})^2+(1-b'_{ij})^2}$

In otherwords, concentration of an IFM is defined by$CON(A) = A^2$.

The following theorem is now straight forward.

Theorem 9: For an IFM A, $CON(A) \leq A$.

Theorem 10: Let $A = (\langle a_{ij}, a'_{ij}\rangle), B = (\langle b_{ij}, b'_{ij}\rangle), C = (\langle c_{ij}, c'_{ij}\rangle)$ and $D = (\langle d_{ij}, d'_{ij}\rangle)$ be IFMs of same size. if $A \geq C, B \geq D$then $A \oplus_\varepsilon C \geq B \oplus_\varepsilon D, A \otimes_\varepsilon C \geq B \otimes_\varepsilon D$.

Proof: Let$A \geq C, B \geq D, i.e., a_{ij} \geq c_{ij}, a'_{ij} \leq c'_{ij}, b_{ij} \geq d_{ij}$ and $b'_{ij} \leq d'_{ij}$.

Then $(a_{ij}-c_{ij})(1-b_{ij}d_{ij}) + (b_{ij}-d_{ij})(1-a_{ij}c_{ij}) \geq 0$

$$i.e., \frac{a_{ij}+b_{ij}}{1+a_{ij}b_{ij}} \geq \frac{c_{ij}+d_{ij}}{1+c_{ij}d_{ij}}$$

and$a'_{ij}b'_{ij}(1-c'_{ij}) \leq c'_{ij}d'_{ij}(1-a'_{ij}), a'_{ij}b'_{ij}(1-d'_{ij}) \leq c'_{ij}d'_{ij}(1-b'_{ij}) \Leftrightarrow$

$$a'_{ij}b'_{ij}(1-c'_{ij}) + a'_{ij}b'_{ij}(1-d'_{ij}) \leq c'_{ij}d'_{ij}(1-a'_{ij}) + c'_{ij}d'_{ij}(1-b'_{ij}) \Leftrightarrow$$
$$a'_{ij}b'_{ij}(1-c'_{ij}) + a'_{ij}b'_{ij}(1-d'_{ij}) + a'_{ij}b'_{ij}c'_{ij}d'_{ij} \leq c'_{ij}d'_{ij}(1-a'_{ij}) + c'_{ij}d'_{ij}(1-b'_{ij}) + a'_{ij}b'_{ij}c'_{ij}d'_{ij} \Leftrightarrow$$
$$a'_{ij}b'_{ij}\left(1+(1-c'_{ij})(1-d'_{ij})\right) \leq c'_{ij}d'_{ij}\left(1+(1-a'_{ij})(1-b'_{ij})\right) \Leftrightarrow$$
$$\frac{a'_{ij}b'_{ij}}{\left(1+(1-a'_{ij})(1-b'_{ij})\right)} \leq \frac{c'_{ij}d'_{ij}}{\left(1+(1-c'_{ij})(1-d'_{ij})\right)}$$

By definition we have, $A \oplus_\varepsilon C \geq B \oplus_\varepsilon D$.

Similarly we have$\frac{a_{ij}b_{ij}}{1+(1-a_{ij})(1-b_{ij})} \geq \frac{c_{ij}d_{ij}}{1+(1-c_{ij})(1-d_{ij})}$

,$\frac{a'_{ij}b'_{ij}}{(1+a'_{ij}b'_{ij})} \leq \frac{c'_{ij}d'_{ij}}{(1+c'_{ij}d'_{ij})}$

By definition we have, $A \otimes_\varepsilon C \geq B \otimes_\varepsilon D$.

Theorem 11: Let $A = (\langle a_{ij}, a'_{ij}\rangle) and B = (\langle b_{ij}, b'_{ij}\rangle)$be any two intuitionistic fuzzy matrices (IFMs) of same order then

(i) $(A^c \to B) \oplus_\varepsilon (A \to B^c)^c = A \oplus_\varepsilon B$and

(ii)$(A^c \to B) \otimes_\varepsilon (A \to B^c)^c = A \otimes_\varepsilon B$.

Proof :

Let $A = (a_{ij}, a'_{ij}) and B = (b_{ij}, b'_{ij})$be two intuitionistic fuzzy matricesof same order. Also $A^c = (a'_{ij}, a_{ij})$ and $B^c = (b'_{ij}, b_{ij})$.

$$(A^c \to B) = \left(\max(a_{ij}, b_{ij}), \min(a'_{ij}, b'_{ij})\right)$$

$$(A \to B^c) = \left(\max(a'_{ij}, b'_{ij}), \min(a_{ij}, b_{ij})\right)\ (A \to B^c)^c = \left(\min(a_{ij}, b_{ij}), \max(a'_{ij}, b'_{ij})\right)$$

$$(A^c \to B) \oplus_\varepsilon (A \to B^c)^c$$
$$= \left(\max(a_{ij}, b_{ij}), \min(a'_{ij}, b'_{ij})\right) \oplus_\varepsilon \left(\min(a_{ij}, b_{ij}), \max(a'_{ij}, b'_{ij})\right)$$

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

$$= \left[\frac{\max(a_{ij}, b_{ij}) + \min(a_{ij}, b_{ij})}{1 + \max(a_{ij}, b_{ij})\min(a_{ij}, b_{ij})}, \frac{\max(a'_{ij}, b'_{ij})\min(a'_{ij}, b'_{ij})}{1 + \left(1 - \max(a'_{ij}, b'_{ij})\right)\left(1 - \min(a'_{ij}, b'_{ij})\right)}\right]$$

Also for any two real numbers c and d, we have max(c, d)+min(c, d)= c + d and max(c, d) min(c, d)= cd.

$$= \left[\frac{a_{ij} + b_{ij}}{1 + a_{ij} b_{ij}}, \frac{a'_{ij} b'_{ij}}{1 + (1 - \max(a'_{ij}, b'_{ij}) - \min(a'_{ij}, b'_{ij}) + \max(a'_{ij}, b'_{ij})\min(a'_{ij}, b'_{ij}))}\right]$$

$$= \left[\frac{a_{ij} + b_{ij}}{1 + a_{ij} b_{ij}}, \frac{a'_{ij} b'_{ij}}{1 + (1 - (a'_{ij} + b'_{ij}) + a'_{ij} b'_{ij})}\right]$$

$$= \left[\frac{a_{ij} + b_{ij}}{1 + a_{ij} b_{ij}}, \frac{a'_{ij} b'_{ij}}{1 + (1 - a'_{ij})(1 - b'_{ij})}\right]$$

$=A \oplus_\varepsilon B$

$(A^c \rightarrow B) \otimes_\varepsilon (A \rightarrow B^c)^c$

$$= \left(\max(a_{ij}, b_{ij}), \min(a'_{ij}, b'_{ij})\right) \otimes_\varepsilon \left(\min(a_{ij}, b_{ij}), \max(a'_{ij}, b'_{ij})\right)$$

$$= \left[\frac{\max(a_{ij}, b_{ij})\min(a_{ij}, b_{ij})}{1 + \left(1 - \max(a_{ij}, b_{ij})\right)\left(1 - \min(a_{ij}, b_{ij})\right)}, \frac{\max(a'_{ij}, b'_{ij}) + \min(a'_{ij}, b'_{ij})}{1 + \left(\max(a'_{ij}, b'_{ij})\right)\left(\min(a'_{ij}, b'_{ij})\right)}\right]$$

$$= \left[\frac{(a_{ij}, b_{ij})}{1 + (1 - \max(a_{ij}, b_{ij}) - \min(a_{ij}, b_{ij}) + \max(a_{ij}, b_{ij})\min(a_{ij}, b_{ij}))}, \frac{a'_{ij} + b'_{ij}}{1 + a'_{ij}, b'_{ij}}\right]$$

$$= \left[\frac{a_{ij} b_{ij}}{1 + (1 - (a_{ij} + b_{ij}) + a_{ij} b_{ij})}, \frac{a'_{ij} + b'_{ij}}{1 + a'_{ij} b'_{ij}}\right]$$

$$= \left[\frac{a_{ij} b_{ij}}{1 + (1 - a_{ij})(1 - b_{ij})}, \frac{a'_{ij} + b'_{ij}}{1 + a'_{ij} b'_{ij}}\right] = A \otimes_\varepsilon B.$$

IV. CONCLUSION

In this article, the Einstein operations onintuitionistic fuzzy matrices have been defined and various properties are presented. Further, we proved De Morgan's laws for these operations over complement andNecessity and Possibility operators.

REFERENCES

[1] K.Atanassov, Instuitionistic fuzzy sets. Fuzzy sets and systems 20(1)(1986),87-96.
[2] K.H.Kim and F.W.Roush, Generalized fuzzy matrices.Fuzzy sets and systems .Vol.4(1980)293-315.
[3] K.W.Oh and W.Bandler, Properties of fuzzy implication operators, Int.J.ofApproxi-mate Reasoning, Vol.1(13)(1987)273-285.
[4] M.Z.Ragab and E.G.Emam, On the min-max composition of fuzzy matrices, Fuzzy Sets and Systems, Vol.75,(1995)83-92.
[5] T.M. Selvarajan, S. Sriram and R. S.Ramya, Einstein OperationsofFuzzy Matrices.International Journal of Engineering & Technology, [S.l.], v. 7,n. 4.36, p. 813-816, dec. 2018. ISSN 2227-524X.
[6] A.K.Shyamal and M.Pal, Two new operators on fuzzy matrices, J.Appl.Math.Computing,Vol.15(1-2),(2004),91-107.
[7] M.G.Thomason, Convergence of powers of a fuzzy matrix J.Math.Anal.Appl.57(1977) 476-480.
[8] W.Wang and X.Liu, Intuitionistic Fuzzy Information Aggregation usingEinstein operations, IEEE Transactions on Fuzzy systems, Vol.20(5)(2012)923-938.
[9] L.A.Zadeh, Fuzzy sets,Information and Control Vol.8(1965)338-353.