Dynamics of Strong Prey and weak Predator with Hvrvesting of Prey

G A L satyavathi ,Paparao.A.V,K.Sobhan Babu

Abstract : In the paper we tend to investigate the harvesting effort of prey species in prey predator model with a strong Prey (N_1),and a weak Predators (N_2). The system is delineated by a system of ordinary differential equations. The Four equilibrium points are known. Local stability analysis is mentioned at every equilibrium points. Global stability is studied by constructing appropriate Lyapunov's function. Additional Numerical simulation is performed and known the parameters for the system that becomes stable and unstable. Further the results are compared with harvesting effort and while not effort on the prey species. The Harvesting efforts of the prey species with catch ability constant and effort are known that unstable system becomes stable therefore the harvesting effort stabilizes the system.

Index Terms: Prey, Predator, Harvesting, Equilibrium points, Local stability, Global stability.
Mathematics classification: 34-XX

I. INTRODUCTION

The stability analysis of prey –predator models are wide studied Lokta [1] and Volterra [2], and w.j.Meyer [3]. The intensive study during this fascinating field of population dynamics was dealt by P.Colinvaux [4], Freedman [5]. The detail study of various models in Ecology and medicine are studied by Kapur [6, 7]. Normally prey predator models, predators possess ability to attack prey from the genesis of the ecological models .The stability analysis of such models are quite attention grabbing.

K.Gopal swamy [8] studied the global asymptotic stability in vloterra's population systems. V.SreeHari Rao [9] briefly explains the prey, predator dynamics with different potentials. Paparao [10] mentioned the prey-predator model linear cover for prey population and alternative food for predator. Papraro [11] studied the dynamics of strong prey and weak predator. Here we tend to take into account of prey population with harvesting effort wherever predator possesses less ability to attack prey. During this paper we tend to study the harvesting effort of the prey. The proposed model is explained by the couple of non linear differential equations. The Four equilibrium points are identified and local stability at the equilibrium points is mentioned. The global stability is studied by construction a suitable Lyapunov's function. The results are compared with the harvesting effort and while not with harvesting effort on the prey species with appropriate illustrations in support of stability analysis of the system. The harvesting effort on prey species plays a big role in during which the unstable system while not the harvesting of prey species because of its strong ability to oppose the predator species becomes stable.

II. MATHEMATICAL MODEL

The system of equations for the proposed model is

$$\frac{dN_1}{dt} = a_1 N_1 - \alpha_{11} N_1^2 - \frac{\alpha_{12} N_1 N_2}{1 + kN_1} - qEN_1$$

$$\frac{dN_2}{dt} = a_2 N_2 - \alpha_{22} N_2{}^2 + \frac{\alpha_{21} N_1 N_2}{1 + kN_2} \tag{2.1}$$

III. EQUILIBRIUM STATES

Parameter	Description
N_1, N_2	Prey predator populations respectively
a_1, a_2	Growth rates of prey and predator
$\alpha_{ii}(i=1,2)$	Interspecies completion rates (negative values)
K	Proportional constant
α_{12}	Interaction coefficient of prey with predator(negative value)
α_{21}	Interaction coefficient of predator with prey
q	Catch ability coefficient
E	Effort

We obtain the equilibrium points by equating $\frac{dN_i}{dt} = 0$ i =1, 2

The Extinct state

$$E_1: \overline{N}_1 = 0, \overline{N}_2 = 0 \tag{3.1}$$

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Semi Extinct

A. only one species is survive

E_2 : $\overline{N_1} = 0$, $\overline{N_2} = \dfrac{a_2}{\alpha_{22}}$ (3.2)

E_3: $\overline{N_1} = \dfrac{a_1 - qE}{\alpha_{11}}$, $\overline{N_2} = 0$ (3.3)

B. Two species are survive

E_5 : $\overline{N_1} = (a_1 - qE) \pm \dfrac{\sqrt{(a_1 - qE)^2 - 4\lambda\alpha_{11}\alpha_{12}}}{2\alpha_{11}}$,

$$\overline{N_2} = a_2 \pm \frac{\sqrt{a_2^2 + 4\lambda\alpha_{11}\alpha_{21}}}{2\alpha_{22}} \quad (3.4)$$

Where λ is called survival of the species

The possible equilibrium for this case are expressed in terms of λ with many possibilities

IV. LOCAL STABILITY ANALYSIS

The community matrix for the system of equations (2.1) is given by

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

where

$$J_{11} = \frac{\partial f_1}{\partial N_1}, J_{12} = \frac{\partial f_1}{\partial N_2},$$
$$J_{21} = \frac{\partial f_2}{\partial N_1}, J_{22} = \frac{\partial f_2}{\partial N_2} \quad (4.1)$$

Here

$$f_1(N_1, N_2) = a_1 N_1 - \alpha_{11} N_1^2 - \frac{\alpha_{12} N_1 N_2}{1 + kN_1} - qEN_1,$$
$$f_2(N_1, N_2) = a_2 N_2 - \alpha_{22} N_2{}^2 + \frac{\alpha_{21} N_1 N_2}{1 + kN_2} \quad (4.2)$$

Calculate the Jacobean matrix i.e.

$$J = \begin{bmatrix} -\dfrac{\alpha_{11} N_1}{k} + \dfrac{\alpha_{12} N_1 N_2}{(1 + kN_1)^2} & -\alpha_{12} N_1 \\ \alpha_{21} N_2 & -\alpha_{22} N_2 \end{bmatrix} \quad (4.3)$$

The characteristic equation is given by det(J-λI)=0 ,
Find the eigen values for the matrix if the all the eigen values are negative, the system becomes stable otherwise unstable. If the eigen values are complex , if the real part of the complex roots are negative , the system becomes stable The system is stable if the Eigen roots of equation (4.3) are negative in case of real roots or negative real parts in case complex roots, otherwise unstable.

Case (i) $E_1(0,0)$ is un stable

Case (ii) A: The characteristic equation for E_2 is $\lambda^2 + \left(\dfrac{\alpha_{11} N_1}{k}\right) = 0$, where roots are purely imaginary makes the system stable.

Case (ii) B: The characteristic equation for E_3 is $\lambda^2 + \alpha_{11} N_2 = 0$, where roots are purely imaginary makes the system stable

Case (iii): co-existing case E_4: The characteristic equation is given by $a\lambda^2 + b\lambda + c = 0$ (4.4)

where a=1,

$$b = \left(\frac{\alpha_{12} N_1 N_2}{(1 + kN_1)^2} - \frac{\alpha_{11} N_1}{k} - \alpha_{22} N_2 \right),$$
$$c = \left(\frac{\alpha_{12}\alpha_{21} N_1 N_2}{(1 + kN_1)^2} - \frac{\alpha_{12}\alpha_{22} N_1 N_2^2}{(1 + kN_1)^2} + \frac{\alpha_{11}\alpha_{22} N_1 N_2}{k} \right) \quad (4.5)$$

The system is stable if the following conditions are satisfied .

$$\text{if } \left(\frac{\alpha_{12} N_1 N_2}{(1 + kN_1)^2} < \frac{\alpha_{11} N_1}{k} + \alpha_{22} N_2 \right) \&$$
$$\left(\frac{\alpha_{12}\alpha_{21} N_1 N_2}{(1 + kN_1)^2} > \frac{\alpha_{12}\alpha_{22} N_1 N_2^2}{(1 + kN_1)^2} + \frac{\alpha_{11}\alpha_{22} N_1 N_2}{k} \right) \quad (4.6)$$

Therefore E_4 is locally asymptotically stable

V. GLOBAL STABILITY ANALYSIS

Theorem: The axial equilibrium point $E_4\left(\overline{N_1}, \overline{N_2}\right)$ is globally asymptotically stable

Proof: Choose the following lyapunov function

$$V\left(\overline{N_1}, \overline{N_2}\right) = (N_1 - \overline{N_1}) - \overline{N_1}\log\left(\frac{N_1}{\overline{N_1}}\right) + l_1\left[(N_2 - \overline{N_2}) - \overline{N_2}\log\left(\frac{N_2}{\overline{N_2}}\right)\right] \quad (5.1)$$

The time derivate of V along the solutions of equations 2.1 is

$$\frac{dV}{dt} = \frac{dN_1}{dt}\left[1 - \frac{\overline{N_1}}{N_1}\right] + l_1 \frac{dN_2}{dt}\left[1 - \frac{\overline{N_2}}{N_2}\right]$$

(5.2)

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$$= [N_1 - \overline{N_1}]\left[(a_1 - qE) - \alpha_{11}N_1 - \frac{\alpha_{12}N_2}{1+kN_1}\right] +$$

$$l_1[N_2 - \overline{N_2}]\left[a_2 - \alpha_{22}N_2 + \frac{\alpha_{21}N_1}{1+kN_1}\right] \quad (5.3)$$

By proper choice of

$$a_1 = \alpha_{11}\overline{N_1} + \frac{\alpha_{12}\overline{N_2}}{1+k\overline{N_1}} + qE, a_2 = \alpha_{22}\overline{N_2} - \frac{\alpha_{21}\overline{N_1}}{1+k\overline{N_1}} \& l_1 =$$

We get $\frac{dV}{dt} = -\alpha_{11}\left(N_1 - \overline{N_1}\right)^2 - \frac{\alpha_{12}\alpha_{22}}{\alpha_{21}}\left(N_2 - \overline{N_2}\right)^2$ (5.4)

Since $\frac{dV}{dt} < 0$ the system is globally asymptotically stable at axial equilibrium point

VI. NUMERICAL EXAMPLE

Example1: Let $a_1 = 1$, $a_2 = 1$, $k=0.5$, $\alpha_{11}=0.1$, $\alpha_{12}=0.1$, $\alpha_{21}=0.05$, $\alpha_{22}=0.5$, $N_1 = 25$, $N_2 = 10$.

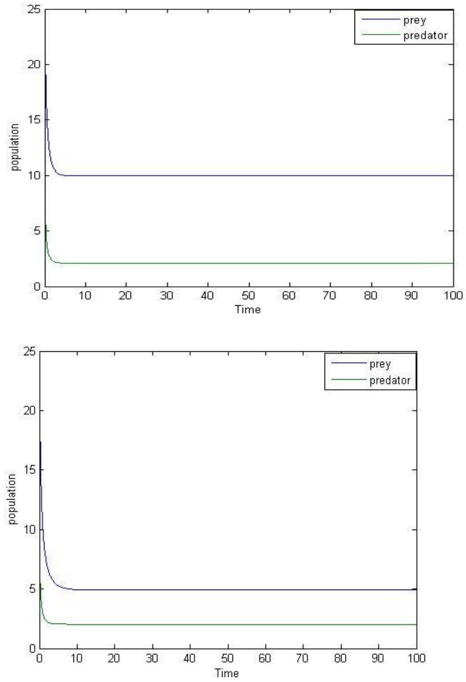

Fig 1 (a)

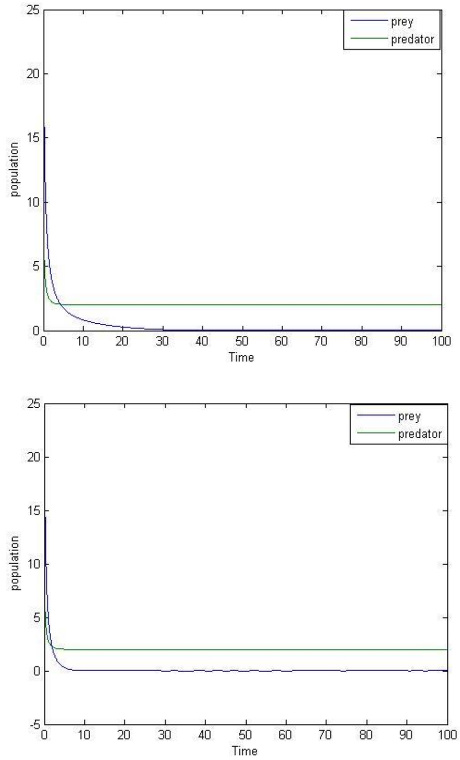

Fig 1 (c)

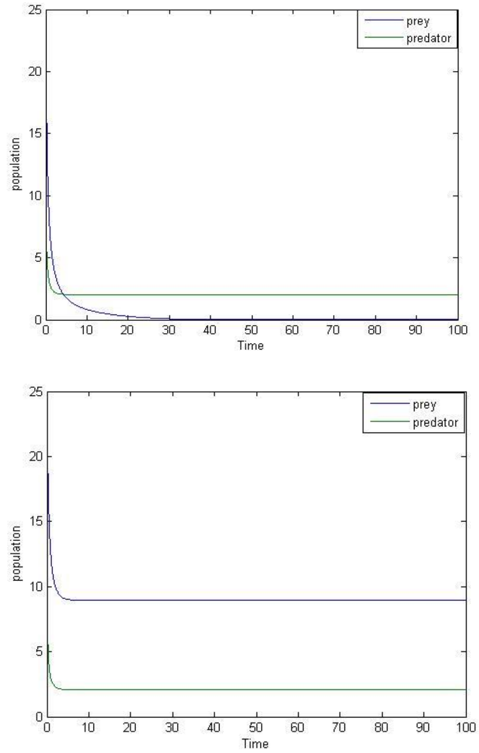

Fig 1 (e)

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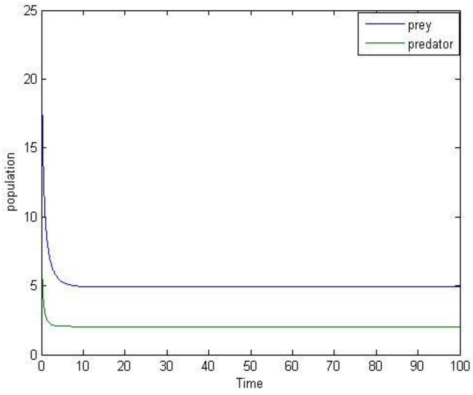


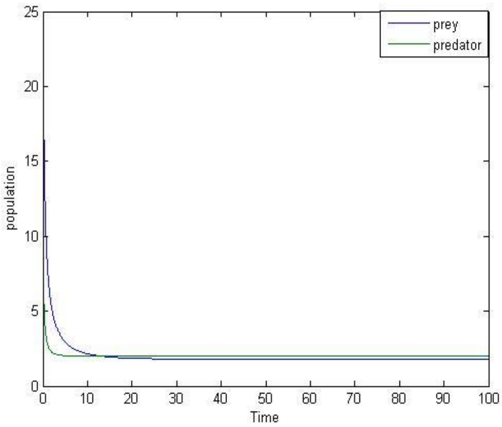


Fig 1 (g)

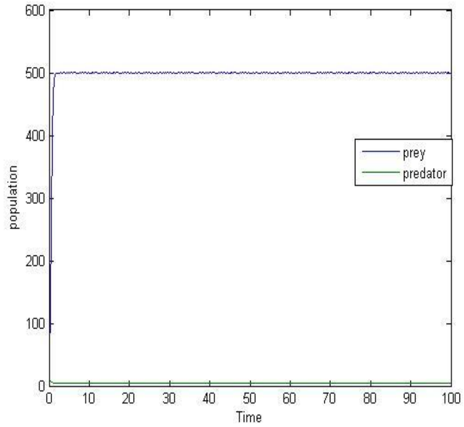
Fig 1(a) : shows the time series variation of the model (2.1) while no harvest effort on the Prey species . The system is stable and converging to the equilibrium point E(10,2)

Fig 1(b) to 1(h) shows The variation of Prey and predator populations with respect to time (t) with different harvesting efforts and catch ability coefficients are shown below .

Fig 1(b) : with q=0.1 , E = 5 . The system is stable and quenching to the equilibrium point E(5,2)
Fig 1(c) : with q=0.2 , E = 5 . The system is stable and quenching to the equilibrium point E(0,2)
Fig 1(d) : with q=0.3 , E = 5. The system is stable and quenching to the equilibrium point E(0,2)
Fig 1(e) : with q=0.1, E = 10. The system is stable and quenching to the equilibrium point E(0,2)
Fig 1(f) : with q=0.1, E = 1. The system is stable and quenching to the equilibrium point E(9,2)
Fig 1(g) : with q=0.5, E = 1. The system is stable and quenching to the equilibrium point E(5,2)
Fig 1(h) : with q=0.8, E = 1. Stable system and quenching to the equilibrium point E(2,2)

The harvesting effort E = 10 with q = 0.1 and E = 5 with q = 0.2, 0.3 makes the system stable and the prey population is extinct. The different efforts are stabilizes the systems.

Example 2: Let a_1=5, a_2=0.5, k =50 α_{11}=0.01, α_{12}=0.1, α_{21}=0.1, α_{22}=0.2, N_1 =30, N_2 =15.



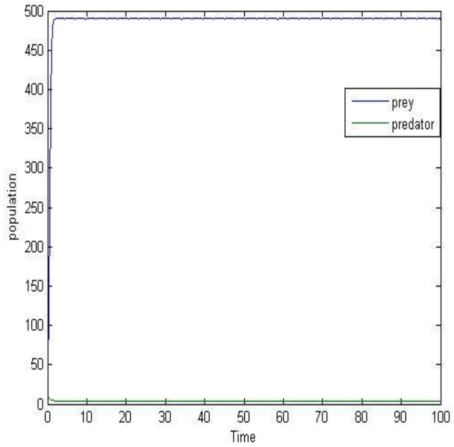
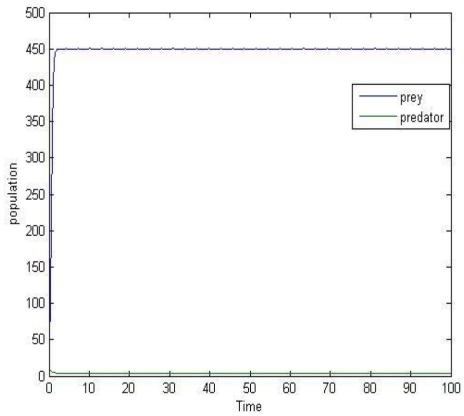


Fig 2 (a)



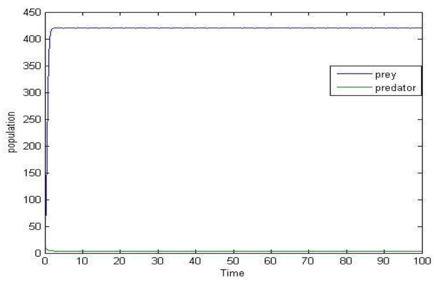
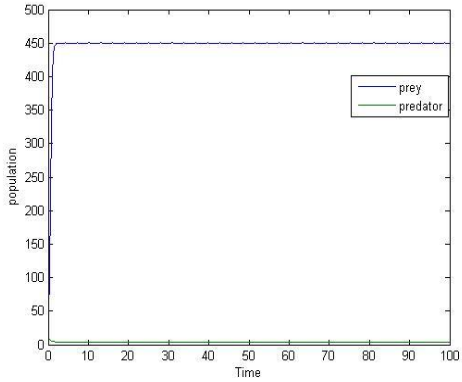


Fig 2 (c)



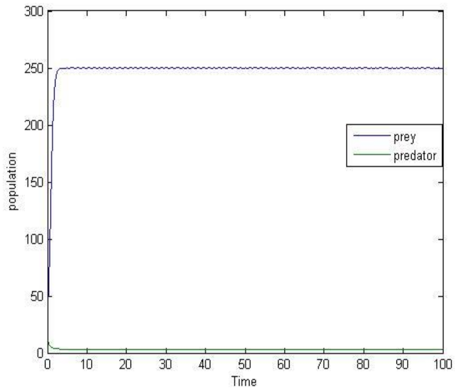
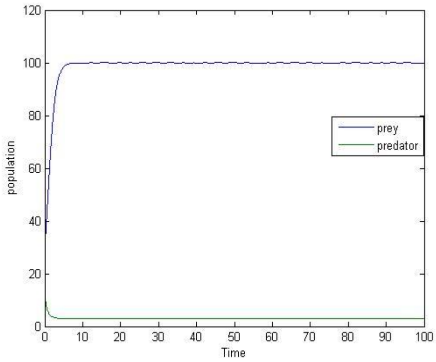
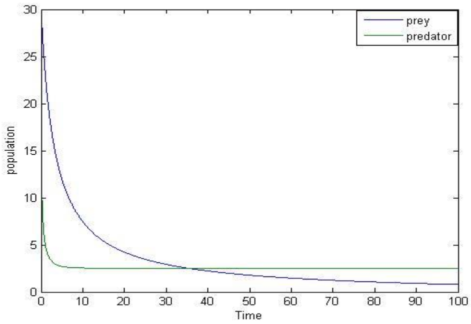


Fig 2 (e)





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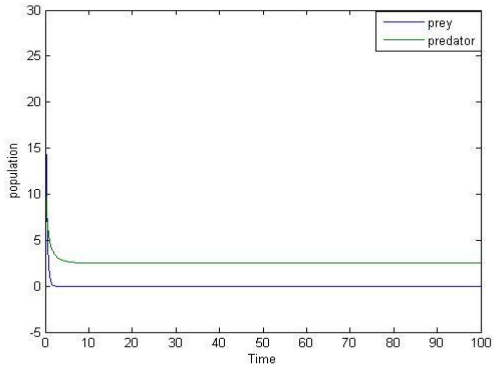

Fig 2(I)

Fig 2(a) : Time series plot while no harvesting effort on the prey species. The system is un stable since the prey population is increasing from its initial strength to 500.

Fig 2(b) to 2(h) shows Time series plot of the model (2.1) The variation of Prey and predator populations with respect to time (t)
with different harvesting efforts and catch ability coefficients are shown below .

Fig 2(b) : with q=0.1 , E = 1 . The system is unstable, and the prey population strength is reduced to 490 and predator population is 4

Fig 2(f)

Fig 2(c) : with q=0.5 , E = 1 . The system is unstable, and the prey population strength is reduced to 450and predator population is 4

Fig 2(d) : with q=0.8 , E = 1. The system is unstable, and the prey population strength is reduced to 420 and predator population is 4

Fig 2(e) : with q=0.1, E = 5. The system is unstable, and the prey population strength is reduced to 450 and predator population is 4.

Fig 2(f) : with q=0.5, E = 5. The system is unstable, and the prey population strength is reduced to 250 and predator population is 4

Fig 2(g) : with q=0.8, E = 5. The system is unstable, and the prey population strength is reduced to 100 and predator population is 3

Fig 2(h) : with q=0.5, E = 10. The system is stable and quenching to the equilibrium point E(1,3)

Fig 2(I) : with q=0.8, E = 10. The system is stable and quenching to the equilibrium point E(0,3) i.e prey population extinct

The harvesting effort for q = 0.5 and E = 10, the unstable system becomes stable, hence forth harvesting effort stabilizes the system. Even for q = 0.8, the system stabilizes at the predator population 3 and prey population extinct for E = 10 .Hence for the above set of parameters the harvesting effort is significant.

Fig 2(h)

Example 3 : Let a_1=1.5, a_2=2, k =5, α_{11}=0.18, α_{12}=0.15, α_{21}=0.17, α_{22}=0.13, N_1 =20, N_2 =35.

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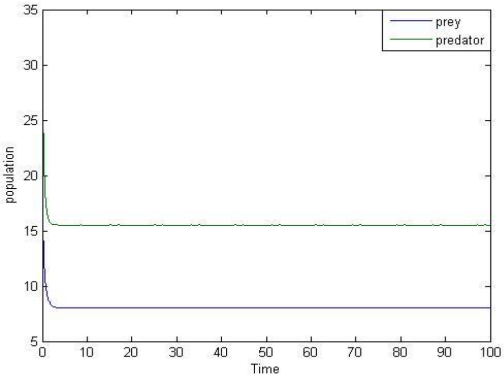
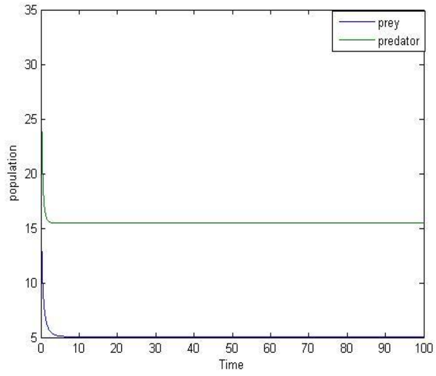
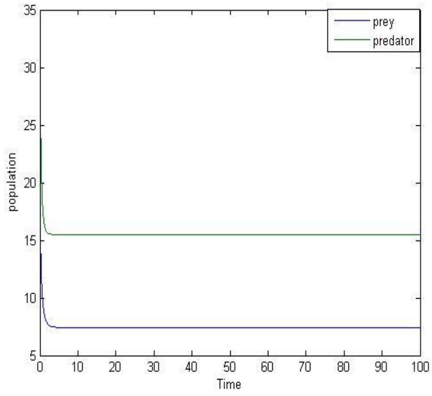






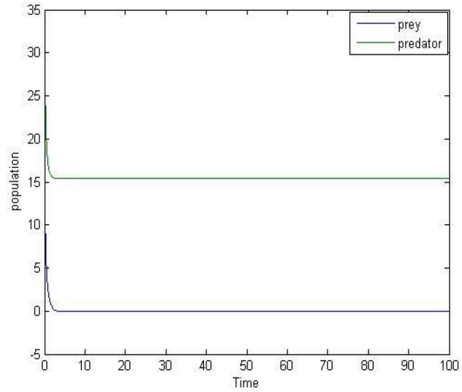
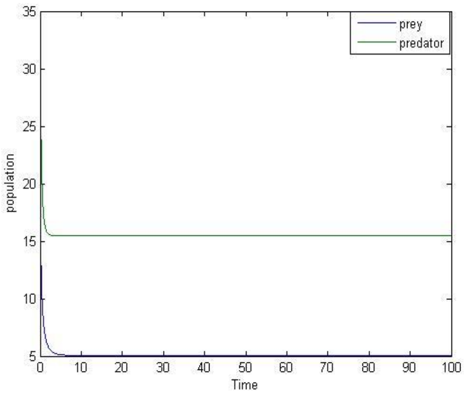
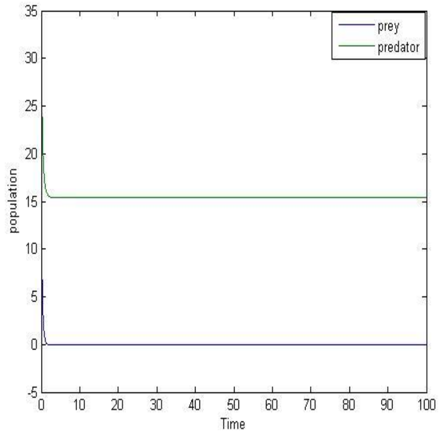


Fig 3(a)

Fig 3(b)

Fig 3 (e)





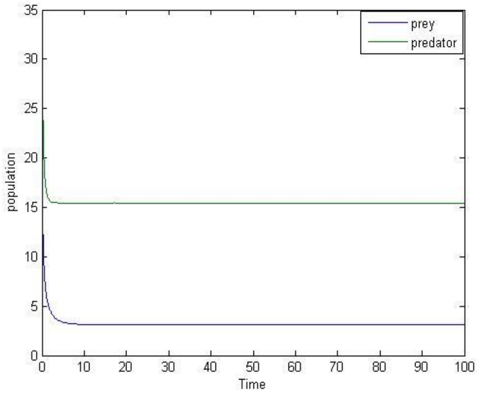


Fig 3 (c)

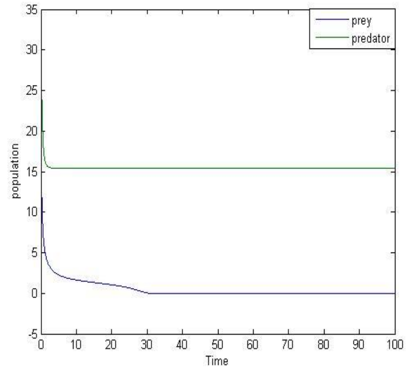
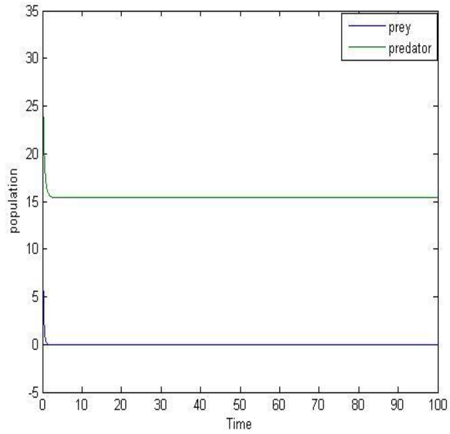


Fig 3 (g)



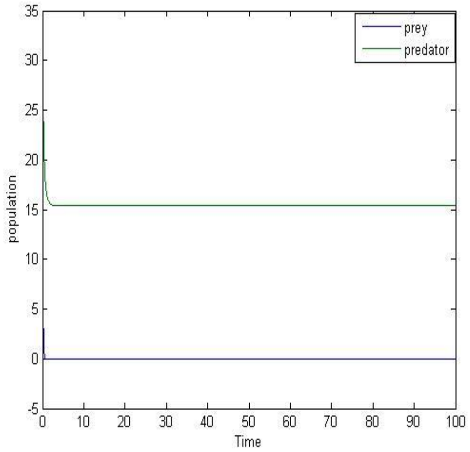


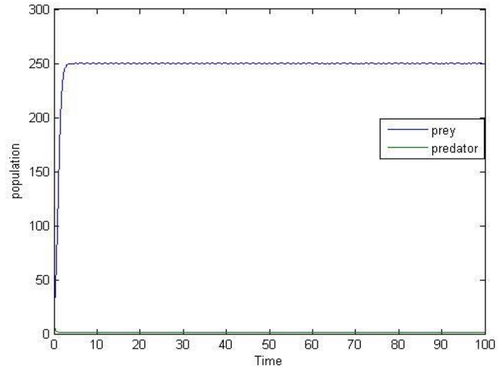
Fig 3 (I)

Fig 3(a) : Time series plot while no harvesting effort on the prey species .The system is stable and converging to the equilibrium point E(8,16)
Fig 3(b) to 3(J) shows time series plot of the model (2.1) with different harvesting efforts and catch ability coefficients are shown below .

Fig 3(b) : with q=0.1 , E = 1 . The system is stable, and converging to stable fixed point E(7,16)
Fig 3(c) : with q=0.5 , E = 1 . The system is stable, and converging to stable fixed point E(5,15)
Fig 3(d) : with q=0.8 , E = 1. The system is stable, and converging to stable fixed point E(5,15)
Fig 3(e) : with q=0.1 , E = 5 . The system is stable, and converging to stable fixed point E(5,15)
Fig 3(f) : with q=0.5 , E = 5 . The system is stable, and converging to stable fixed point E(0,15)
Fig 3(g) : with q=0.8 , E = 5. The system is stable, and converging to stable fixed point E(0,15)
Fig 3(h) : with q=0.1 , E = 10 . The system is stable, and converging to stable fixed point E(0,15)
Fig 3(I) : with q=0.5 , E = 10 . The system is stable, and converging to stable fixed point E(0,15)
Fig 3(J) : with q=0.8 , E = 10. The system is stable, and converging to stable fixed point E(0,15)

Harvesting effort for these suitable parameters is studied and system still exhibits stable behaviour. The catch ability constant q > 0.5 and Effort E = 5 and E = 10 , the prey population is extinct and predator population also stabilizes at 15.

Example 4 : Let a_1=2.5, a_2=0.5, k =5 α_{11}=0.01, α_{12}=0.01, α_{21}=0.01, α_{22}=0.7, N_1 =20, N_2 =25.



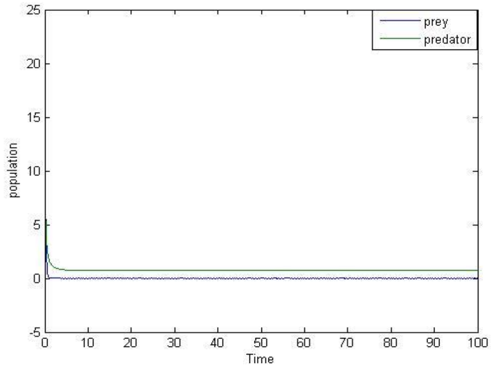
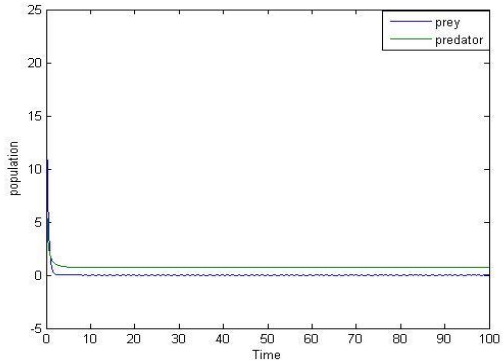


Fig 3(J) Fig 4 (a)



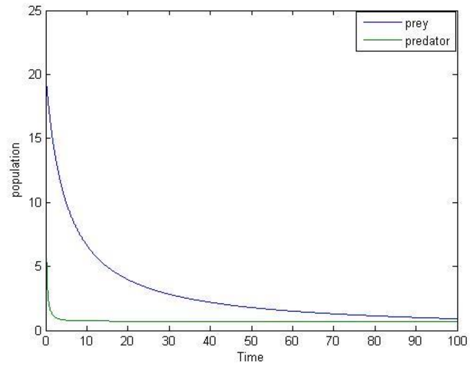


Fig 4 (c)

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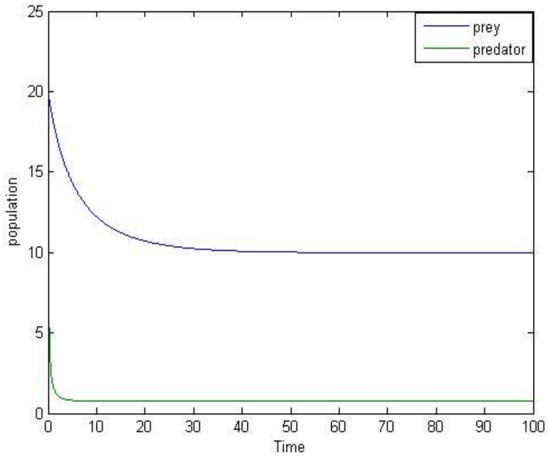

Fig 4 (e)

Fig 4(a): Time series plot while no harvesting effort on the prey species. The system is un stable since the prey population is increasing from its initial strength to 250

Fig 4(b) to 4(e) shows The variation of Prey and predator populations with respect to time (t)
with different harvesting efforts and catch ability coefficients are shown below .

Fig 4(b) : with q=0.8 , E = 10. The system is stable, and prey population is extinct converging to stable fixed point E(0,1)

Fig 4(c) : with q=0.5 , E = 10. The system is stable, and prey population is extinct converging to stable fixed point E(0,1)

Fig 4(d) : with q=0.25 , E = 10. The system is stable, and converging to stable fixed point E(1,1)

Fig 4(e) : with q=0.4 , E = 6. The system is stable, and converging to stable fixed point E(10,1)

The harvesting effort stabilizes the system for E = 10, 6 with q = 0.25, 0.4 respectively and for
E = 10, q with 0.5 or 0.8, the prey population extinct and predator population stabilizes to 1, hence the harvesting efforts are significant to make unstable systems to stable ones.

VII. CONCLUSION

In this paper we contemplate a two species ecological model supported prey predator interactions with a strong rey and weak predator. Additionally that harvest of prey spices is considered. The doable equilibrium points are known and local stability of the system is discussed. The extinct state is unstable and remaing two cases are asymptotically stable. At the axial equlibrium point the system is conditionally stable. Global stablity analysis is studied by lyapunov's function and shown that the system is globally asymptotically stable . The solutions of the system of equations delineated diagrammatically with appropriate examples. The dynamics of model for the different parametric values are illustrated for which the system becomes stable or unstable . The harvesting effort with different efforts and catch ability coefficients are studied whereever the stable system additional stablizes and unstable system becomes stable with mat lab simulation. The effort for extinct conditions for prey predator populations are derived and additionally identified the acceptable parameters of the model that is globally asymptotically stable.

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