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# Design of Master-Slave Coupling for Chaotic Synchronization of Maps

**A. K. Das, M. K. Mandal**

***Abstract:*** *--- In this paper, we analyse the master-slave coupling for synchronization of two chaotic systems through nonlinear control law. The proposed coupling technique is illustrated using the 1D Logistic map, 2D and 3D Henon maps. The MATLAB simulation results confirm the validity of the designed coupling law. Experimental realization of the targeted coherent dynamics is presented in this paper using 1D Logistic map.*

***Keywords:*** *Chaos, Synchronization, Maps, Master-slave coupling.*

## 1. Introduction

The study of chaotic synchronization was first reported by Yamada and Fujisaka in 1983 [1]. But the problem of chaotic synchronization became popular in the scientific community after the pioneering work of Pecora and Carroll [2] and suggested application in secure communication. The important applications of synchronized chaos are being explored in secure communication and cryptography [3-8] using different types of coupling. In this regard, we need to mention different synchronization techniques like complete synchronization (CS) [2, 9, 10], phase synchronization (PS) [11], antiphase synchronization (APS) [12-14], lag synchronization (LS) [15], generalized synchronization (GS) [16, 17], partial synchronization [18] and predictive synchronization[19], which are studied to understand the dynamics of the systems and synchronous phenomena. Depending upon the nature of the problem under consideration and field of applications, either continuous or discrete dynamical systems [3, 4] are used as individual oscillators for investigation. The interactions among the oscillators of an integrated system are studied by introducing different types of coupling strategy, for example, diffusive coupling, delay coupling, open-plus-closed-loop (OPCL) coupling, master-slave coupling, etc. The diffusive coupling (nearest-neighbour coupling) has been first introduced in a diffusion like process of a physical systems [20, 21]. To understand the phenomena of a system of interacting oscillators, the diffusive coupling can be used to model the oscillator systems for investigation. The delay coupling is used to study the interaction among the interactive units, which involves certain delay with each other, of a dynamical system. Time delay system is an important topic of research in the field of electronic communication systems [22-24], neuroscience [25], etc. On the other hand, OPCL coupling [8, 26] is generally used to investigate the synchronized phenomena of a coupled system under parameter mismatch cases. Pecora and Carrol [2] popularised the chaotic synchronization by introducing master-slave coupling in electronic system. In this technique, two systems are considered with same sets of equation and parameter values. At least one state variable from the master system force or drive the slave system to follow the behaviour of the master system, due to this reason master system is known as driver system and slave system is known as response system. The above coupling mechanism have been extensively used in the literature to investigate the synchronous phenomena of continuous systems governed by differential equations. However, these coupling strategy has not been studied in case of discrete systems extensively so far.

Therefore in this article, we investigate the master-slave coupling for synchronization of two discrete systems. We have used unidirectional coupling between two chaotic maps through nonlinear technique to study the synchronized behaviour of the concerned maps. The same coupling strategy is tested in 1D logistic map, 2D and 3D Henon maps through numerical simulation. The experimental verification of the numerical observations has been done by hardware electronic circuit realization of 1D Logistic map. The proposed strategy may be applied in secure data communication in electronic [27] as well as in optical domain [28] by parameter modulation or chaos masking techniques.

## 2. Synchronization of chaotic maps

In this section, we describe a theorem [27, 29] for the design of master-slave synchronization rule for chaotic maps.

**Theorem 1:** If $e_{i+1} = A_i e_i + U_i e_{i,}\ \forall i \geq 0$, $|eig(A_i + U_i)| = |eig(B)| < 1$. Then $\|e_i\| \rightarrow 0$, as $i \rightarrow \infty, \forall e_0 \in R^i$. The theorem states that the equilibrium 0, of the error system $e_{i+1}$, is globally asymptotically stable if and only if all eigenvalues of $B = A_i + U_i$ have magnitude less than one. In the above theorem the symbol $|\ \ |$ denote the magnitude of the eigenvalues of a matrix and the symbol $\|\ \ \|$ denote the euclidian norm.

### *2.1 Synchronization of 1D Logistic map*

We start our discussion with the one dimensional Logistic map as master system. The map is given by

$$x_{i+1} = a x_i (1 - x_i), \tag{1}$$

where the state variable $x_i$ lies between 0 and 1 and the parameter $a$ is a positive number between 0 and 4. The master system drives another identical slave system which is given by

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$$X_{i+1} = aX_i(1 - X_i) + u_i. \quad (2)$$

Here $u_i$ is nonlinear control factor that can synchronize $X_i$ with $x_i$. The error between maser and slave system is defined by the following equation.

$$e_i = X_i - x_i. \quad (3)$$

Therefore, the difference error between master and slave systems is given by

$$e_{i+1} = X_{i+1} - x_{i+1} = a(1 - X_i - x_i)e_i + u_i. \quad (4)$$

Rewriting equation (4) to fit the form of the theorem 1 we obtained

$$e_{i+1} = a(1 - X_i - x_i)e_i + U_i e_i \quad (5)$$

Where $A_i = a(1 - X_i - x_i)$ and $u_i = U_i e_i$. Thus according to theorem 1 we get

$$B = A_i + U_i = a(1 - X_i - x_i) + U_i. \quad (6)$$

The magnitude of eigenvalues of equation (6) is less than unity. Since, $B$ in equation (6) is a scalar (1×1 matrix), then by taking the eigenvalue of $B$ is $\alpha$, we can write $U_i = \alpha - a(1 - X_i - x_i)$, where $-1 < \alpha < 1$. Therefore the nonlinear control factor $u_i$ is given by

$$u_i = U_i e_i = (\alpha - a(1 - X_i - x_i))e_i \quad (7)$$

Therefore, the slave system in this case takes the form

$$X_{i+1} = aX_i(1 - X_i) + (\alpha - a(1 - X_i - x_i))e_i \quad (8)$$

Here $\alpha = 0$ indicates the fastest synchronization between master and slave system and synchronization get delayed with the increasing values of $\alpha$ in both positive and negative direction. The system does not synchronize when the magnitude of $\alpha \geq 1$.

We iterate the master-slave systems (1) and (8) in the chaotic regime of the Logistic map using MATLAB software. We have observed synchronized dynamics of the master-slave system for different values of $\alpha$. Fig. 1 shows the iteration results for $a = 3.99$ and $\alpha = 0.31$. In Fig. 1a, after first few iteration, the superposition of the corresponding iterates of the master and slave system indicates complete synchronization. Fig. 1b shows the steady state pot of $X_i$versus $x_i$ that confirms complete synchronization.

*2.2 Synchronization of 2D Henon map*

Now in second example, we consider 2D Henon map as master system is given by

$$x_{i+1} = 1 - ax_i^2 + by_i \quad (9a)$$

$$y_{i+1} = x_i \quad (9b)$$

The corresponding slave system is defined as

$$X_{i+1} = 1 - aX_i^2 + bY_i + u_{1i} \quad (10a)$$

$$Y_{i+1} = X_i + u_{2i}. \quad (10b)$$

The difference error between master and slave systems is defined as

$$e_{1i+1} = X_{i+1} - x_{i+1} = -a(X_i + x_i)e_{1i} + be_{2i} + u_{1i} \quad (11a)$$

$$e_{2i+1} = Y_{i+1} - y_{i+1} = e_{1i} + u_{2i} \quad (11b)$$

In 2×2 matrix notation the error $e_{i+1} = A_i e_i + U_i e_i$ may be expressed as

$$e_{i+1} = \begin{pmatrix} a_{11i} & a_{12i} \\ a_{21i} & a_{22i} \end{pmatrix}\begin{pmatrix} e_{1i} \\ e_{2i} \end{pmatrix} + \begin{pmatrix} u_{11i} & u_{12i} \\ u_{21i} & u_{22i} \end{pmatrix}\begin{pmatrix} e_{1i} \\ e_{2i} \end{pmatrix} \quad (12)$$

where $A_i = \begin{pmatrix} a_{11i} & a_{12i} \\ a_{21i} & a_{22i} \end{pmatrix}$, $U_i = \begin{pmatrix} u_{11i} & u_{12i} \\ u_{21i} & u_{22i} \end{pmatrix}$, $e_i = \begin{pmatrix} e_{1i} \\ e_{2i} \end{pmatrix}$ and nonlinear control factors are $u_{1i} = u_{11i}e_{1i} + u_{12i}e_{2i}$ and $u_{2i} = u_{21i}e_{1i} + u_{22i}e_{2i}$. Rewriting equation (11) in matrix form we obtain

$$e_{i+1} = \begin{pmatrix} -a(X_i + x_i) & b \\ 1 & 0 \end{pmatrix}\begin{pmatrix} e_{1i} \\ e_{2i} \end{pmatrix} + \begin{pmatrix} u_{11i} & u_{12i} \\ u_{21i} & u_{22i} \end{pmatrix}\begin{pmatrix} e_{1i} \\ e_{2i} \end{pmatrix}. \quad (13)$$

Comparing equations (12) and (13) we obtain

$$B = A_i + U_i = \begin{pmatrix} -a(X_i + x_i) + u_{11i} & b + u_{12i} \\ 1 + u_{21i} & u_{22i} \end{pmatrix} \quad (14)$$

According to theorem 1, we choose $u_{11i} = a(X_i + x_i)$ and $u_{12i} = u_{21i} = u_{22i} = 0$ to obtain a constant matrix $B$. Therefore, the nonlinear control factors can be obtained as $u_{1i} = a(X_i + x_i)e_{1i}$ and $u_{2i} = 0$. The constant matrix $B = \begin{pmatrix} 0 & b \\ 1 & 0 \end{pmatrix}$. The magnitude of the eigenvalues $\alpha$ of matrix $B$ must be less than unity to obtain complete synchronization between master and slave systems. This condition provided $\alpha = \pm\sqrt{b} < 1$. The final expression of slave system is given below by incorporating nonlinear control law in equation (10) as

$$X_{i+1} = 1 - aX_i^2 + bY_i + a(X_i + x_i)e_{1i} \quad (15a)$$

$$Y_{i+1} = X_i. \quad (15b)$$

The numerical iteration of the master-slave systems (9) and (15) is done in the chaotic regime to illustrate the complete synchronization. Fig. 2a shows the successive iterates of the variables $x_i$ and $X_i$ of master and slave system and it revels the state of complete synchronization for $a = 1.3$ and $b = 0.32$. It is further confirmed by plotting the variables $X_i$ versus $x_i$ in steady state condition in Fig. 2b. Similar representations of variables $y_i$ and $Y_i$ are given in Figs. 2c and 2d.

*2.3 Synchronization of 3D Henon map*

Next a similar exercise is done by taking more complex system 3D Henon map. The master system is defined as

$$x_{i+1} = 1 - ax_i^2 + by_i \quad (16a)$$

$$y_{i+1} = x_i + z_i \quad (16b)$$

$$z_{i+1} = x_i. \quad (16c)$$

The corresponding slave system in this case takes the form

$$X_{i+1} = 1 - aX_i^2 + bY_i + u_{1i} \quad (17a)$$

$$Y_{i+1} = X_i + Z_i + u_{2i} \quad (17b)$$

$$Z_{i+1} = X_i + u_{3i}. \quad (17c)$$

The difference error between these two systems can be represented as


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$$e_{1i+1} = X_{i+1} - x_{i+1} = -a(X_i + x_i)e_{1i} + be_{2i} + u_{1i} \quad (18a)$$

$$e_{2i+1} = Y_{i+1} - y_{i+1} = e_{1i} + e_{3i} + u_{2i} \quad (18b)$$

$$e_{3i+1} = Z_{i+1} - z_i = e_{1i} + u_{3i} \quad (18c)$$

In 3×3 matrix notation the error $e_{i+1} = A_i e_i + U_i e_{i,}$ can be written as

$$e_{i+1} = \begin{pmatrix} a_{11i} & a_{12i} & a_{13i} \\ a_{21i} & a_{22i} & a_{23i} \\ a_{31i} & a_{32i} & a_{33i} \end{pmatrix} e_i + \begin{pmatrix} u_{11i} & u_{12i} & u_{13i} \\ u_{21i} & u_{22i} & u_{23i} \\ u_{31i} & u_{32i} & u_{33i} \end{pmatrix} e_i, \quad (19)$$

where $e_i = \begin{pmatrix} e_{1i} \\ e_{2i} \\ e_{3i} \end{pmatrix}$ and nonlinear control laws are $u_{1i} = u_{11i}e_{1i} + u_{12i}e_{2i} + u_{13i}e_{3i}$, $u_{2i} = u_{21i}e_{1i} + u_{22i}e_{2i} + u_{23i}e_{3i}$ and $u_{3i} = u_{31i}e_{1i} + u_{32i}e_{2i} + u_{33i}e_{3i}$. Rewriting equation (18) in matrix notation to fit the matrix equation (19) we obtain

$$e_{i+1} = \begin{pmatrix} -a(X_i + x_i) + u_{11i} & b & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} e_i + \begin{pmatrix} u_{11i} & u_{12i} & u_{13i} \\ u_{21i} & u_{22i} & u_{23i} \\ u_{31i} & u_{32i} & u_{33i} \end{pmatrix} e_i. \quad (20)$$

Now observing equations (19) and (20) we can write

$$B = A_i + U_i = \begin{pmatrix} -a(X_i + x_i) + u_{11i} & b + u_{12i} & u_{13i} \\ 1 + u_{21i} & u_{22i} & 1 + u_{23i} \\ 1 + u_{31i} & u_{32i} & u_{33i} \end{pmatrix}. \quad (21)$$

We choose $u_{11i} = a(X_i + x_i)$, $u_{12i} = 0$, $u_{13i} = c$, $u_{21i} = u_{22i} = u_{23i} = u_{31i} = u_{32i} = u_{33i} = 0$, which leads to the control laws $u_{1i} = a(X_i + x_i)e_{1i} + ce_{3i}$ and $u_{2i} = u_{3i} = 0$. Where c is a constant to be determined. Using these control laws we obtain

$$B = \begin{pmatrix} 0 & b & c \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

According to theorem 1, to obtain complete synchronization the magnitude of the eigenvalues ($\alpha$) of matrix $B$ must be less than unity. The eigenvalue equation of matrix $B$ is $\alpha^3 + b\alpha - c\alpha - b = 0$. For simplicity we take $c = b$, which leads to $\alpha = \pm b^{1/3} < 1$. Using nonlinear control law in equation (17) we obtain the final slave system as

$$X_{i+1} = 1 - aX_i^2 + bY_i + a(X_i + x_i)e_{1i} + ce_{3i} \quad (22a)$$

$$Y_{i+1} = X_i + Z_i \quad (22b)$$

$$Z_{i+1} = X_i. \quad (22c)$$

The numerical simulation of the master-slave systems (16) and (22) is shown in Fig. 3 for $a = 1.05$ and $b = 0.33$. Figure 3a shows the successive iterates of the variables $x_i$ (master) and $X_i$ (slave), which proves the complete synchronization. It is again confirmed in Fig. 3b by showing the variation of $X_i$ with respect to $x_i$ in $X_i - x_i$ plane. Similar representations for other variables $(y_i, Y_i)$ and $(z_i, Z_i)$ are also shown in Figs. 3c-3d and Figs. 3e-3f respectively. In this section, we are able to successfully demonstrate the targeted complete synchronization of master-slave systems using 1D Logistic map, 2D and 3D Henon maps.

### 3. Electronic experiment & Results

The present section describes the experimental realization of the above mention master-slave synchronization scheme using 1D Logistic map. We have observed complete synchronization in a hardware experiment in our laboratory. Figure 4 shows the complete experimental circuit diagram for two coupled 1D Logistic maps using nonlinear control law based on master-slave coupling. Commercially available electronic components such as analog multiplier (AD633), operational amplifier (TL082), sample-hold circuit (LF398), resistors and capacitors are used in the circuit. The used multiplier AD633 has an inherent characteristic to scale down the output voltage by a factor of 10, that is, if the input of the multiplier are *A* and *B* then its output will be *AB*/10. Therefore, the state variables $x_i$ and $X_i$ are scaled up to a factor 10 to avoid small signal error and noisy effect, on the other hand the parameter $a$ is also scaled up to a factor of 2.5 for the same reason (because maximum value of $a$ is 4). Therefore, we choose $x_i' = 10x_i$, $X_i{}' = 10X_i$ and $a' = 2.5a$. Using these state variables $(x_i{}', X_i{}')$ and parameter $(a')$ the master-slave systems (equations (1)and (8)) can be modified as

$$\text{Master system:} \; x'_{i+1} = \frac{a'}{25} x'_i (10 - x'_i) \quad (23)$$

$$\text{Slave system:} \; X'_{i+1} = \frac{a'}{25} X'_i (10 - X'_i) + [\alpha - \frac{a'}{25}(10 - X'_i - x'_i)]e'_i. \quad (24)$$

Where $e'_i = X'_i - x'_i$. In electronic circuit we have take $\alpha = 0$ for the fastest synchronization. In Fig. 4a, the first TL082 acts as a subtractor whose inputs are 10, $x'_i$ and its output is $(10 - x'_i)$. Next themultiplier AD633 produces an output $\frac{1}{10} x'_i (10 - x'_i)$, since its inputs are $(10 - x'_i)$ and $x'_i$. Then the multiplier output is passed through a gain adjustment amplifier for four times amplifications. This amplified signalis then passes through the second multiplier circuit to multiply it with the parameter $a'$ to obtain the targeted expression of the master system as mention in equation (23). Finally, the multiplier output is passed through two successive sample-hold circuits to obtain the discrete nature of the state variable $x'_i$. A suitable clock signal is connected to the triggering input of the first sample-hold circuit and the inverted clock signal is used to trigger the second sample-hold circuit to get the require discrete voltage level of the state variable $x'_i$. The slave circuit is similar to that of the master circuit but one extra adder circuit is used here to add the nonlinear control signal $u'_i$ with the output of the sample-hold circuit to obtain required state variable $X'_i$ as shown in Fig. 4b. The circuit diagram of the nonlinear control law is shown in Fig. 4c. The used ICs with actual pin numbers and passive electronic components (resistors and capacitors) including their values are indicated in Fig. 4. Finally, the continuity among the different parts of the circuit is indicated through the state variables and arrow marks.


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The hardware experimental results are recorded from the points $x_i'$ and $X_i'$ marked in the master and slave circuit using Agilent 54641D digital storage oscilloscope. The results are shown in Figs. 5 and 6 for parameter value $a' = 9.17$. Figure 5a shows the complete synchronization between master and slave systems in time series view and Fig. 5b represents the variation of $X_i'$ with respect to $x_i'$ in $X_i' - x_i'$ plane. This data confirms the targeted synchronization between master and slave systems. When nonlinear control law $u_i'$ (Fig. 4c) is not connected to the adder circuit of the slave system (Fig. 4b) the slave system cannot follow the master system, which leads to unsynchronized result as shown in Fig. 6.

## 4. Conclusions

We present a master-slave coupling strategy to achieve targeted synchronization between two chaotic maps through nonlinear control law. The presented coupling strategy is verified through numerical examples using 1D Logistic map, 2D and 3D Henon maps. The speed of synchronization of this coupling method depends on the magnitude of the eigenvalues α of the matrix *B*, synchronization speed increases with the decreasing value of $\alpha$. Since, iteration of maps is much faster than integrating the differential equations, the proposed master-slave coupling strategy can be very useful to study the synchronous dynamics of large chaotic networks. The physical realization of the proposed coupling strategy is illustrated in case of 1D Logistic map.

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## Figures

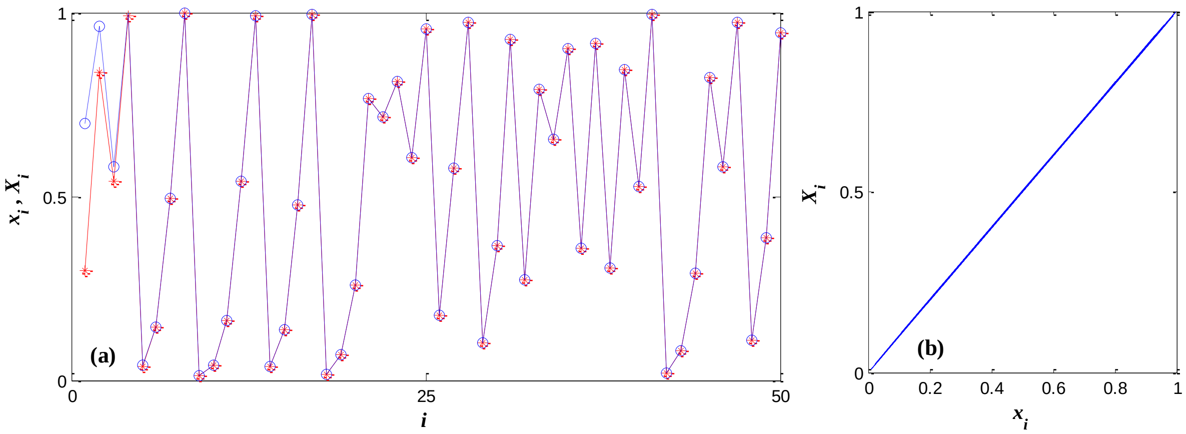



**Fig. 1.** 1D Logistic map. $a = 3.99$ and $\alpha = 0.31$. (a) Successive iterates of the master and slave systems have been shown in red stared solid line and blue circled dashed line respectively. (b) Steady state plot of $X_i$ versus $x_i$.

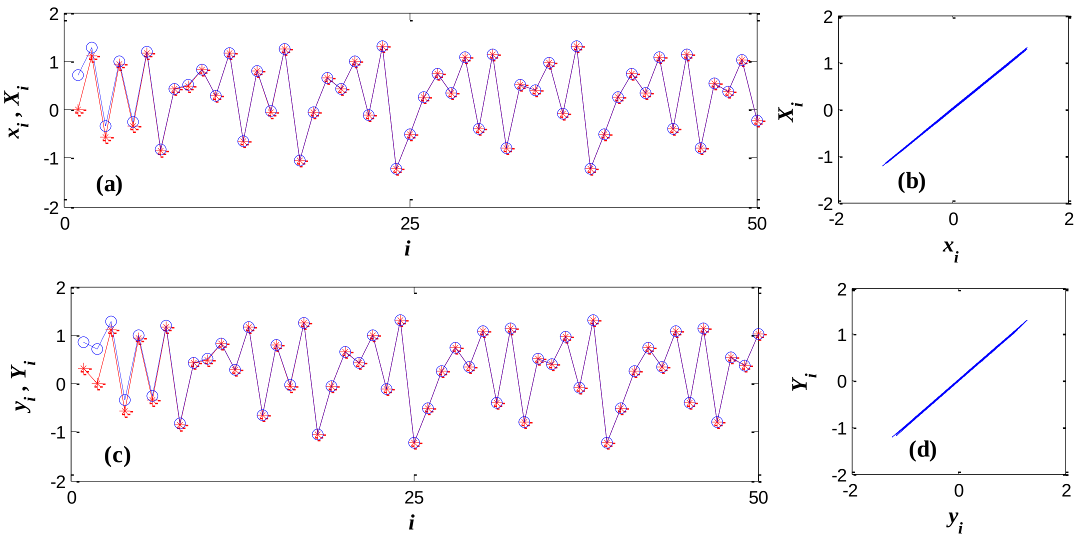



**Fig. 2.** 2D Henon map. $a = 1.3$ and $b = 0.32$. (a) and (c) successive iterates of the master-slave variables $(x_i, X_i)$ and $(y_i, Y_i)$ respectively. Master and slave variables have been shown in red stared solid line and blue circled dashed line respectively. (b) and (d) represent steady state plot of $X_i$ versus $x_i$ and $Y_i$ versus $y_i$ respectively.

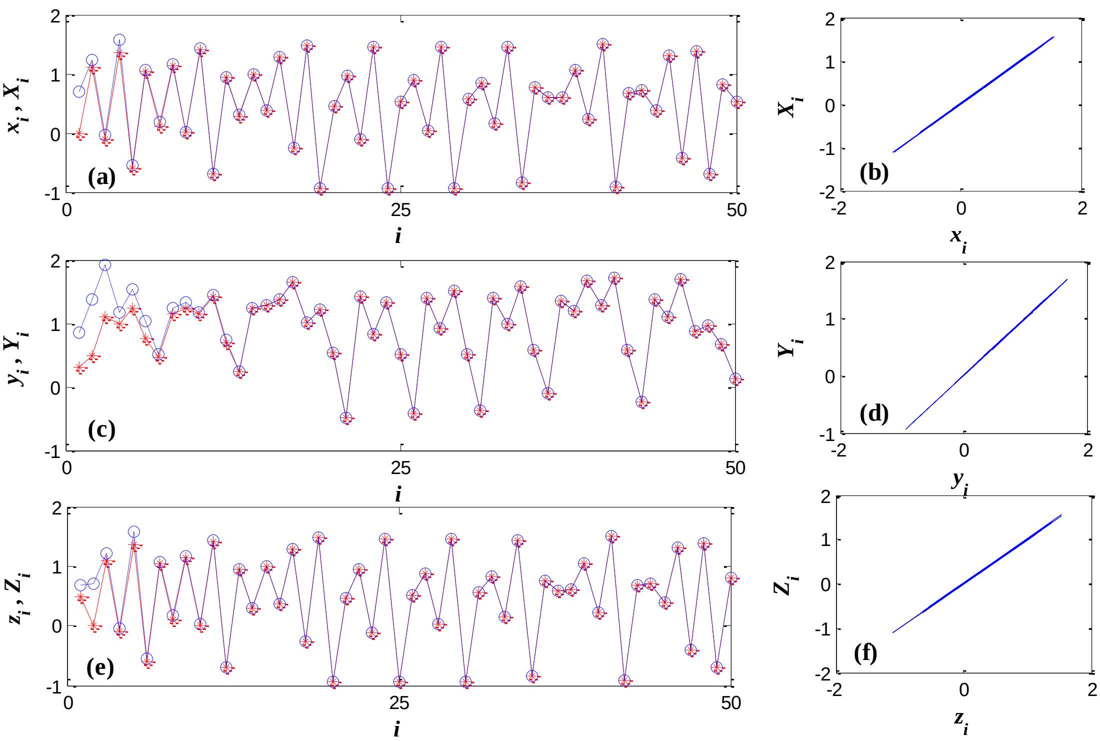
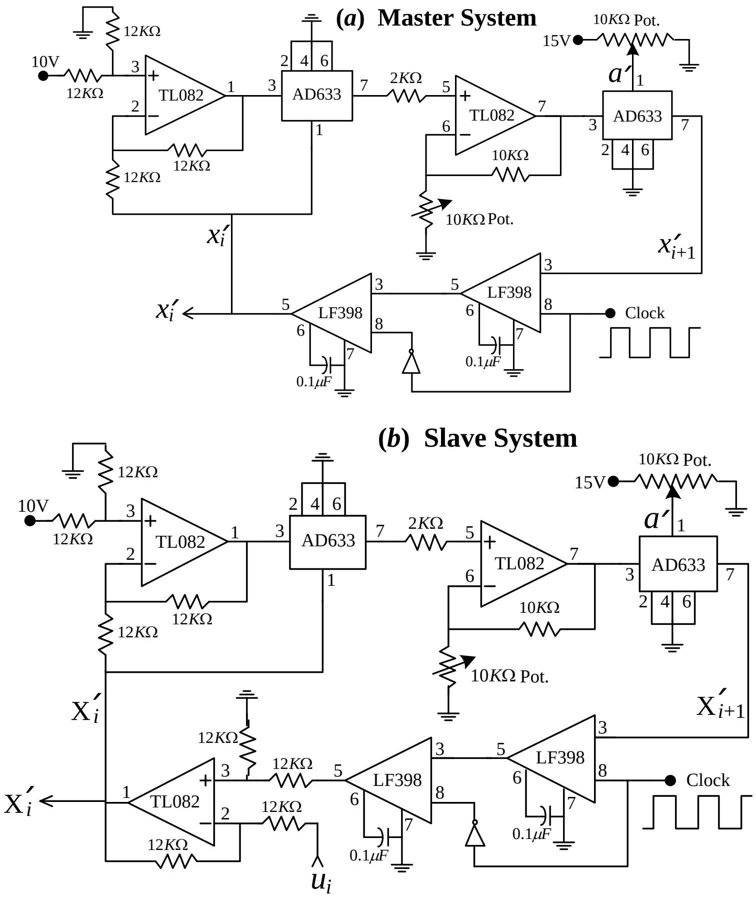



**Fig. 3.** 3DHenon map. $a = 1.05$ and $b = 0.33$. (a) successive iterates of the master variable $(x_i)$ and slave variable $(X_i)$ have been shown in red stared solid line and blue circled dashed line respectively. (b) Represents steady state plot of $X_i$versus $x_i$ in $X_i - x_i$ plane. Similarly, (c) and (e) illustrate the iterates of the variables $(y_i, Y_i)$ and $(z_i, Z_i)$ respectively. (d) and (f) represent the variation of $Y_i$ and $Z_i$ with respect to $y_i$ and $z_i$ respectively.



**(c) Nonlinear Control Law**

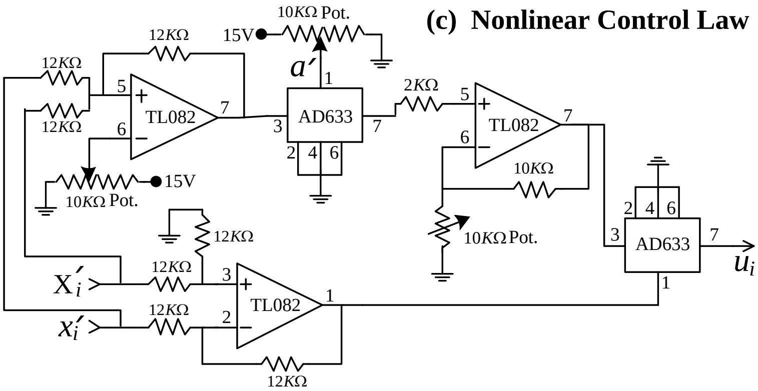



**Fig. 4.** Different parts of the electronic circuit of coupled 1D Logistic map. (a) Master system, (b) Slave system and (c) Nonlinear control law.

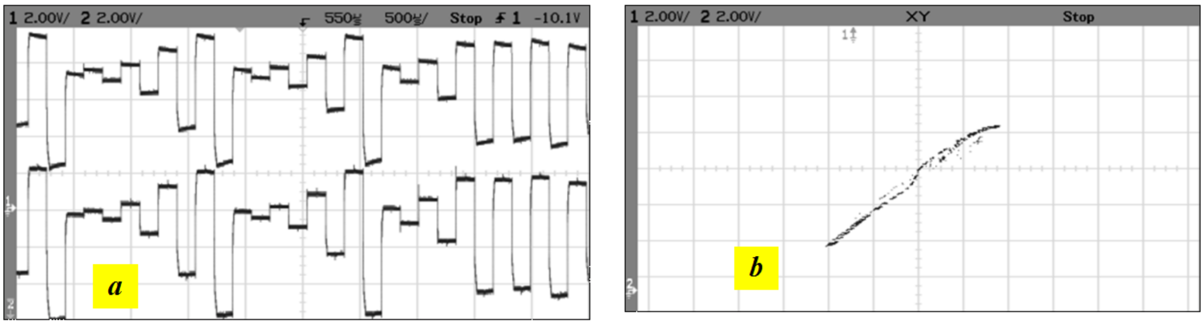



**Fig. 5.** Master-slave synchronization result from hardware experiment. (a) Time series view of state variables $x_i'$ (upper waveform) and $X_i'$ (lower waveform) for $a' = 9.17$. (b) The variation of $X_i'$ verses $x_i'$ in $X_i' - x_i'$ plane.

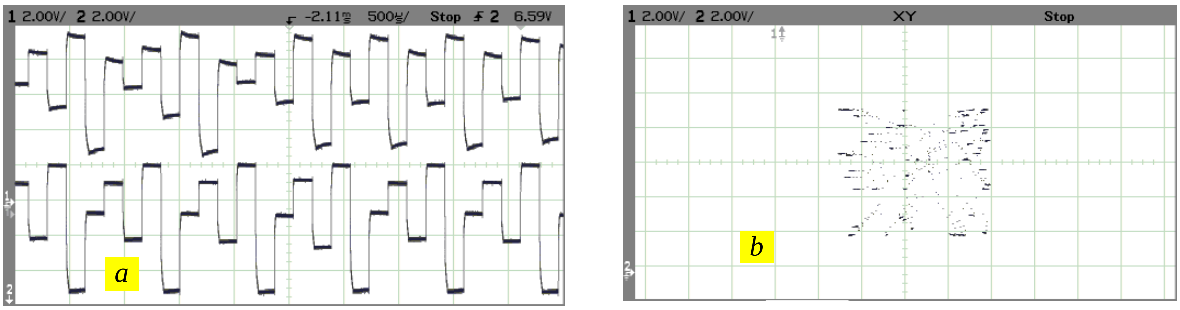



**Fig. 6.** Unsynchronization result from hardware experiment. (a) Time series view of state variables $x_i'$ (upper waveform) and $X_i'$ (lower waveform) for $a' = 9.17$. (b) The variation of $X_i'$ verses $x_i'$ in $X_i' - x_i'$ plane.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