Satellite attitude tracking control using Lyapunov control theory

Mohamed Okasha, Moumen Idres, Alia Ghaffar

***Abstract:** This paper explores attitude control algorithms using thrusters and momentum wheels for satellite tracking manoeuver. The attitude reference trajectory is generated to track a ground object on the rotating Earth and to keep the Sun vector perpendicular to the solar panel to maximize the solar power. The dynamic model is derived based on Newton-Euler approach and Modified Rodrigues Parameters (MRPs) are used to represent attitude kinematics. The control algorithms are derived based on the Lyapunov stability control theory to ensure the asymptotic stability of the control law. MATLAB software package is used as a simulation environment to develop and test the performance of the proposed tracking control algorithms using a Graphical User Interface (GUI) tool that can be expanded for future development.*

***Keywords:** Attitude tracking; Lyapunov stability; Satellite dynamics and control.*

I. INTRODUCTION

Attitude Determination and Control System (ADCS) is used to orient the satellite payload in space and or to observe an object with certain accuracy [1-3]. For example, Hubble Space Telescope (HST) is pointed to observe astronomical objects with very high accuracy. The HST antenna is accurately pointed to a ground station to focus the radio beam and to reduce the power requirements while the HST solar panel is rotated such that the Sun direction becomes perpendicular to maximize the generated solar power [4-6]. Therefore, the ADCS designer must design the system to meet the pointing and accuracy requirements. Design trades are necessary to select the ADCS sensors, actuators and control algorithms based on the satellite mission requirements. In this paper, momentum wheels and thrusters are investigated as actuators for attitude tracking maneuvers. The control algorithms are derived based on Lyapunov control theory and reference trajectory is generated similar to HST mission. The payload is pointed to view and track a particular target on ground, and to generate maximum power by rotating around the payload boresight axis to orient the solar panel perpendicular to the Sun direction.

In literature, Hanblai and Shaube et al. investigate ideal reference trajectories for target tracking [7-9]. In Hablani's ideal tracking, the final angular velocity of the target is not zero and the payload axis is initially facing the zenith direction. The tracking commands utilizes 2-1-3 Euler angle sequence. Shaube et al. computes the tracking commands in terms of a Modified Rodrigues Parameters (MRPs) because it is close to be optimal and less in computation. In addition to target tracking, Kalweit computes ideal attitude commands for Sun tracking. The payload initially facing the Nadir direction. The ideal solar rotation angle is computed to reorient the payload about the yaw axis so that the solar panel normal direction points to the Sun [10]. In this paper, both target tracking and Sun tracking constraints are considered to generate the ideal reference tracking commands.

Schaub et al. and Hall et al use Lyapunov control theory to develop nonlinear tracking controllers. To guarantee global asymptotic stability, they choose the Lyapunov function and drive the control law such that the Lyapunov function derivative is negative definite. A virtual rigid body with the same properties as the real satellite is used to derive the ideal tracking trajectory. Similar controllers are presented in this paper to track the desired virtual trajectories using thrusters and momentum wheels. Thrusters are used to perform large and coarse attitude maneuvers while Momentum wheels provide fine attitude maneuvers to eliminate the tracking errors [4-6]. Attitude kinematics are described using MRPs because they are minimal set and their singularities can be efficiently avoided [11]. MATLAB software package is used as a simulation environment to develop and test the performance of the proposed tracking control algorithms using a Graphical User Interface (GUI) tool that can be expanded for future development [12].

This papers are organizes as follow: Section 1 gives an introduction; Section 2 presents dynamic and kinematics model with definition of reference frames an attitude description; Section 3 derives the ideal reference trajectory for target alignment and Sun tracking including attitude, angular velocity and angular acceleration commands. The development of Lyapunov attitude controller are presented in Section 4; Section 5 shows the simulation results and the controller performance followed by a conclusion in Section 5.

II. DYNAMICS MODEL

The rotational equations motion for satellite with N momentum wheels, in body frame, can be written as [4-6].

$$\begin{aligned} \dot{h}_b &= h_b^{\times} J^{-1}\left(h_b - A h_a\right) + g_e \\ \dot{h}_a &= g_a \\ h_b &= I\omega_b + A I_s \omega_s \end{aligned} \tag{1}$$

where h_b is the satellite angular momentum vector, I is the 3×3 moment of inertia matrix of the entire satellite, I_s is the $N\times N$ wheels axial moment of inertia matrix, A is the $3\times N$ matrix containing the axial unit vectors of the momentum wheels, h_a is the $N\times1$ matrix of the axial angular momentum

Revised Manuscript Received on March 10, 2019.

Mohamed Okasha, Department of Mechanical Engineering, International Islamic University Malaysia, PO Box 10, 50728 Kuala Lumpur, Malaysia (Email: mokasha@iium.edu.my)

Moumen Idres, Department of Mechanical Engineering, International Islamic University Malaysia, PO Box 10, 50728 Kuala Lumpur, Malaysia

Alia Ghaffar, Department of Mechanical Engineering, International Islamic University Malaysia, PO Box 10, 50728 Kuala Lumpur, Malaysia

of the wheels, g_e is the 3×1 matrix of the external torques that include both environmental disturbances and control torques, g_a is the $N\times 1$ matrix of the internal torques applied to momentum wheels, ω_b is the 3×1 angular velocity matrix of the body frame expressed in the inertial frame, ω_s is the $N\times 1$ axial angular velocity matrix of the momentum wheels with respect to the body, and J is the inertia-like matrix defined as

$$J = I - AI_s A^T \tag{2}$$

The MRPs parameters are used to represent kinematics as

$$\dot{\sigma} = G(\sigma)\,\omega_b$$
$$G(\sigma) = \frac{1}{2}\left(II - \sigma^{x} + \sigma\sigma^{T} - \frac{1+\sigma^{T}\sigma}{2} II \right) \tag{3}$$

where ω_b is the body angular velocity, σ is the MRP vector and II is 3×3 the identity matrix.

The MRP vector is defined in terms of Euler axis and angles as

$$\sigma = \vec{e}\tan\frac{\phi}{4} \tag{4}$$

where $\vec{e}$ is the Euler principle axis and is φ the rotation angle. The MRP attitude representation has the advantage of being well define for the entire range of rotation and its singularities can be avoided using MRP shadow set [5, 11].

The satellite translational equations of motion can be expressed by Cowell's formulation as [13]

$$\ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r} + \vec{a}_p \tag{5}$$

where μ is the Earth gravitational parameter, $\vec{r}$ is satellite position vector in inertial frame and $\vec{a}_p$ is acceleration vector due to disturbances.

As shown in Fig. 1, the following frames are defined. The J2000 geocentric inertial frame, $o\hat{x}_n\hat{y}_n\hat{z}_n$ denoted by subscript n and the local vertical local horizontal (LVLH) frame, $o\hat{x}_l\hat{y}_l\hat{z}_l$ denoted by subscript l. The satellite body frame, $o\hat{x}_b\hat{y}_b\hat{z}_b$ denoted by subscript b, is defined in Fig. 2.

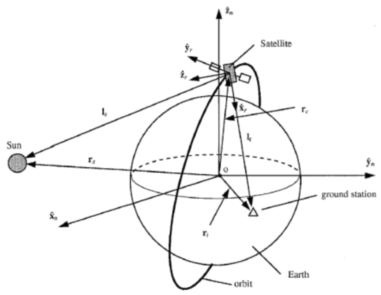

Fig. 1: Satellite mission definition [6]

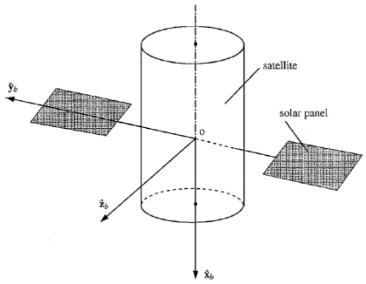

Fig. 2: Satellite body frame [6]

III. REFERENCE TRAJECTORY

In this section, the ideal pointing attitude, ideal angular velocity and ideal acceleration are derived. From Fig. 1, the following vectors are defined

$$\begin{aligned} l_s &= r_s - r_c \\ l_t &= r_t - r_c \end{aligned} \tag{6}$$

where, r_s, r_t and r_c denote the positions of the Sun, the ground station, and the satellite, respectively, and the vectors l_s and l_t denote the vectors from the satellite to the Sun and the ground station. The direction cosine matrix for ideal attitude pointing can be written as

$$R^{ri} = [\hat{x}_r\ \hat{y}_r\ \hat{z}_r]^T \tag{7}$$

where R^{ri} is the rotation matrix from inertial frame to attitude reference frame and $\hat{x}_r$, $\hat{y}_r$, and $\hat{z}_r$ are the unit vectors that can be defined as

$$\hat{x}_r = \frac{l_t}{\ell_t},\ l_t = \ell_t\hat{l}_t,\quad \hat{y}_r = \frac{l_s\times\hat{x}_r}{\|l_s\times\hat{x}_r\|} \text{ and } \hat{z}_r = \hat{x}_r\times\hat{y}_r \tag{8}$$

To achieve the mission, at each moment, the body axis of the satellite should point along the ground station and the axis should be perpendicular to the Sun direction. The value of $\eta_t = \|\hat{l}_t\times\hat{x}_b\|$ is used as a performance condition for ground station tracking and the value of $\eta_s = \hat{l}_s\times\hat{y}_b$ is used as the condition for Sun tracking, i.e, the targets are being tracked if these values approach zero.

The open loop tracking algorithms for ideal angular velocity vector, $\omega_r = [\omega_{rx}\,\omega_{ry}\,\omega_{rz}]^T$, and ideal angular acceleration vector, $\dot{\omega}_r = [\dot{\omega}_{rx}\,\dot{\omega}_{ry}\,\dot{\omega}_{rz}]^T$ are derived similar to references [4-6] as

$$\omega_{rz} = \frac{\dot{l}_t\cdot\hat{y}_r}{\ell_t},\quad \omega_{ry} = -\frac{\dot{l}_t\cdot\hat{z}_r}{\ell_t}$$
$$\omega_{rx} = \frac{\omega_{rz}\cdot l_s\cdot\hat{x}_r - \hat{y}_r\cdot\dot{l}_s}{l_s\cdot\hat{z}_r}$$
$$\dot{\omega}_{rz} = \frac{\ddot{l}_t\cdot\hat{y}_r - 2\omega_{rz}\dfrac{{}^{r}dl_t}{dt}}{\ell_t} - \omega_{rx}\omega_{ry} \tag{9}$$
$$\dot{\omega}_{ry} = -\left(\frac{\ddot{l}_t\cdot\hat{z}_r + 2\omega_{ry}\dfrac{{}^{r}dl_t}{dt}}{\ell_t}\right) + \omega_{rx}\omega_{rz}$$
$$\dot{\omega}_{rx} = \frac{\dot{M}\cdot N - \dot{N}\cdot M}{N^2}$$

The purpose of the control law in next section is to track these ideal desired attitude, R^{ri}, ω_r^{ri}, and $\dot{\omega}_r^{ri}$, in order to track the Sun and the ground target respectively.

IV. TRACKING CONTROLLER DESIGN

In this section, The Lyapunov's direct method is used to derive the feedback controller. The following Lyapunov function is used [6, 7]

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

$$V = \frac{1}{2}\delta\omega^T K\delta\omega + 2k_2 \ln\left(1 + \delta\sigma^T \delta\sigma\right) \quad (10)$$

where $\delta\sigma$ and $\delta\omega$ are the attitude and the angular velocity errors respectively, where MRPs is used to represent the attitude error between the body and the ideal reference trajectory, K and k_2 are positive constants. To derive the control law, the derivate of Eq. (10) is computed as

$$\begin{aligned}\dot{V} = &-[-h_b^\times J^{-1}(h_b - Ah_a) - g_e + Ag_a + J\omega_b \times \delta\omega \\ &+ JR^{br}(\delta\sigma)J^{-1}h_r^\times J^{-1}(h_r - Ah_{ar}) + JR^{br}(\delta\sigma)J^{-1}g_r \\ &- JR^{br}(\delta\sigma)J^{-1}Ag_{ar} - k_2\delta\sigma]^T \delta\omega\end{aligned} \quad (11)$$

where the external torques g_e are defined as

$$g_e = g_g + g_d + g_t \quad (12)$$

the subscript t, g, and d denote the thruster, gravity gradient and disturbance torques respectively. Now, g_t (external thruster torque) or g_a (internal wheel torque) can be selected so that V, becomes negative semi-definite that will ensure the asymptotic stability of the control law.

4.1. Control using Momentum wheels only

In this case, the thrusters will be off (g_t = 0) and the Momentum wheels will provide the desired tracking torque. Equation (11) can be written as

$$\dot{V} = -(Ag_a + z)^T \delta\omega \quad (13)$$

The control torque is selected such that

$$Ag_a = k_1\delta\omega - z \quad (14)$$

this leads to

$$\dot{V} = -k_1\delta\omega^T \delta\omega \le 0 \quad (15)$$

where k_1 is a positive contestant and V becomes negative semi-definite. Then, the wheel control torque that grantees perfect tracking can be written as

$$\begin{aligned}Ag_a = &h_b^\times J^{-1}(h_b - Ah_a) + g_e - J\omega_b \times \delta\omega \\ &- JR^{br}(\delta\sigma)J^{-1}h_r^\times J^{-1}(h_r - Ah_{ar}) - JR^{br}(\delta\sigma)J^{-1}g_r \\ &+ JR^{br}(\delta\sigma)J^{-1}Ag_{ar} + k_1\delta\omega + k_2\delta\sigma\end{aligned} \quad (16)$$

4.2. Control using thrusters only

Thrusters are used for fast and large maneuvers that require high control authority instead of low control authority produced by momentum wheels, in that case

$$Ah_a = 0, \quad Ag_a = 0, \quad Ah_{ar} = 0 \;\; and \;\; Ag_{ar} = 0 \quad (17)$$

then, Eq (11) becomes

$$\dot{V} = -(g_t + z)^T \delta\omega \quad (18)$$

as in momentum wheel case, the thrusters torque is selected as

$$\begin{aligned}g_t = &-h_b^\times J^{-1}h_b - g_d - g_g + J\omega_b \times \delta\omega \\ &+ JR^{br}(\delta\sigma)J^{-1}h_r^\times J^{-1}h_r + JR^{br}(\delta\sigma)J^{-1}g_r - k_1\delta\omega - k_2\delta\sigma\end{aligned} \quad (19)$$

this torque makes V semi-negative definite and guarantees perfect tracking.

V. SIMULATION RESULTS

Simulation are performed with a GUI tool designed in MATLAB software package as in Fig. 3. The control laws, derived in previous section, are used for target maneuvers to track desired attitude of Sun and ground station simultaneously by using thrusters or momentum wheels. Initial conditions of simulations are given in Table 1-5. It includes the orbit initial data, spacecraft physical data, initial attitude data, location of the target to be tracked and finally the controller gains.

In addition to gravity gradient torque, other disturbing torques due to aerodynamic, solar radiation pressure and other environmental factors defined by Eq. (20) are considered in simulation [3, 6].

$$g_d = \begin{bmatrix} 4\times10^{-6} + 2\times10^{-6}\sin(\omega_o t) \\ 6\times10^{-6} + 3\times10^{-6}\sin(\omega_o t) \\ 3\times10^{-6} + 3\times10^{-6}\sin(\omega_o t) \end{bmatrix} N.m \quad (20)$$

where ω_o is the orbital angular velocity.

Fig. 3: GUI Designed Tool for Attitude Tracking

Table 1: Orbit data

Parameter	Value
Semimajor axis	6978.1370 Km
Eccentricity	0.0
Argument of perigee	0.2319^0
Inclination	22.9844^0
RAAN	112.5017^0
True anomaly	42.0117^0

Table 2: Spacecraft physical data

Parameter	Value
Spacecraft moment of inertia (J) [Kg.m^2]	$\begin{bmatrix} 320 & 0 & 0 \\ 0 & 300 & 0 \\ 0 & 0 & 275 \end{bmatrix}$
Wheels moment of inertia (I_s) [Kg.m^2]	$\begin{bmatrix} 14.43 & 0 & 0 & 0 \\ 0 & 14.43 & 0 & 0 \\ 0 & 0 & 14.43 & 0 \\ 0 & 0 & 0 & 14.43 \end{bmatrix}$
Solar panel axis (y_b)	$\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$
Boresight Sensor axis (x_b)	$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$
Wheel orientation (pyramid angle 45^0)	$\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

Table 3: Spacecraft initial conditions

Parameter	Value
$\sigma_b(0)$	$[0.14904 \;\; -0.89515 \;\; 0.42011]$
$\omega_b(0)$ [rad/sec]	$[-0.004 \;\; 0.00854 \;\; 0.0009]$
$\omega_s(0)$ [rad/sec]	$[3.3465 \;\; 3.3465 \;\; 3.3465 \;\; 3.3465]$

Table 4: Target data

Cape Canaveral, Florida	
Parameter	**Value**
Latitude	28.467^0 N
Longitude	80.467^0 W

Table 5: Lyapunov controller gains

Parameter	Value
k_1	27
k_2	24

The results of simulation are presented in Fig. 4-8. Figure 4 shows the time tracking history of angular velocity and MRPs attitude errors. The Lyapunov controller successfully drives these errors asymptotically to zero in 100 seconds. In addition, the controller also shows the Sun tracking condition, η_s and the ground station (at Cape Canaveral, Florida) tracking condition, η_t go to zero after 100 seconds as shown in Figure 5. Figures 7 and 8 show the torque required for performing this tracking maneuver using momentum wheels or thrusters as computed by Eq. (16) and Eq. (19) respectively. Figure 8 verifies the controller stability as the Lyapunov function, V approaching a stable origin ($V \to 0$).

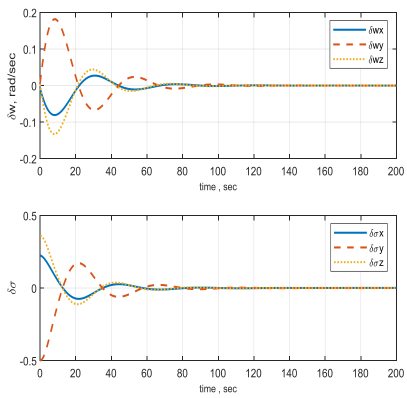

Fig. 4: Time history of $\delta\omega$ and $\delta\sigma$

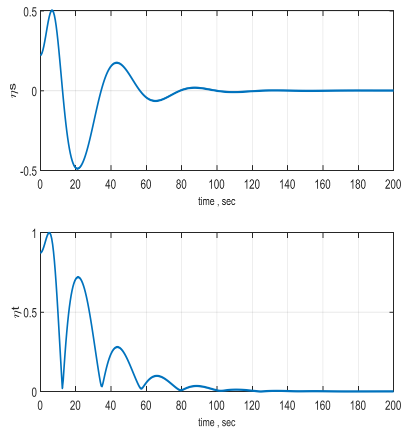

Fig. 5: Sun and Ground Station Tracking Conditions

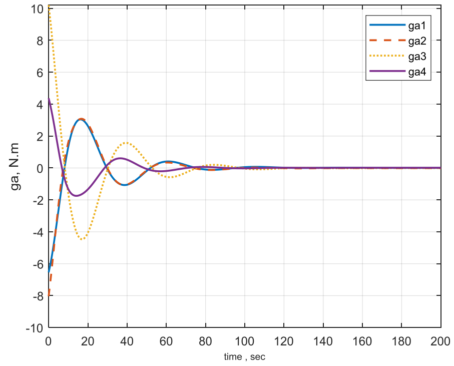

Fig. 6: The momentum Wheel Control Torque

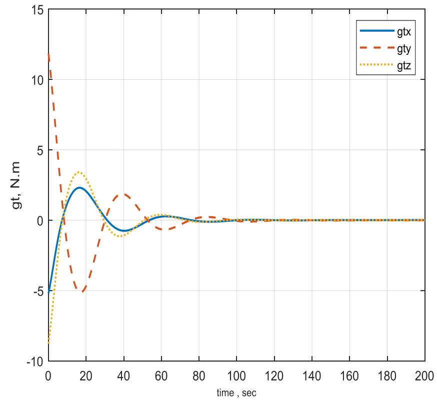

Fig. 7: The Thrusters Control Torque

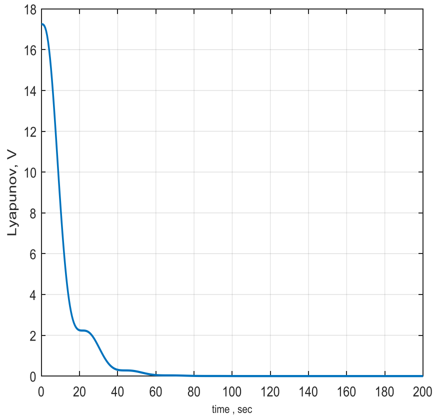

Fig. 8: The Lyapunov Function

VI. CONCLUSION

In this paper, trajectory attitude tracking controllers are developed for a satellite based on Laypunov stability control theory using thrusters and momentum wheels. The tracking trajectory is derived such that the Sun remains perpendicular to the solar panel while tracking a ground object on the rotation Earth. A GUI has been designed in MATLAB to test the performance of the control algorithms. Simulation results demonstrates the efficiency of the proposed algorithms. Future research will investigate deeply the process of selecting the controller gains and to exploit different types of control approaches compared to current method in terms of tracking performance and control efforts.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

ACKNOWLEDGEMENT

This work was supported by Research Initiative Grant Scheme (RIGS17-046-0621) through International Islamic University Malaysia.

REFERENCES

1. Wertz JR (1978), *Spacecraft Attitude Determination and Control*, Springer Netherlands.
2. Hughes PC (1986), *Spacecraft Attitude Dynamics*, John Wiley & Sons, New York.
3. Sidi M (1997), Spacecraft Dynamics and Control: A Practical Engineering Approach, Cambridge Univ. Press, New York.
4. Robert LM (1999), Spacecraft Attitude Tracking Control, Master of Science in Aerospace Engineering, Blacksburg, Virginia.
5. Hall CD, Tsiotras P, and Shen H (1998), Tracking Rigid Body Motion Using Thrusters and Momentum Wheels, Journal of the Astronautical Sciences 50.
6. Tsiotras P, Shen H, and Hall CD (2001), Satellite Attitude Control and Power Tracking with Energy/Momentum Wheels, *Journal of Guidance, Control, and Dynamics* 24, 23-34.
7. Schaub H, Robinett RD, and Junkins JL (1996), Globally Stable Feedback Laws for Near-Minimum-Fuel and Near-Minimum-Time Pointing Maneuvers for a Landmark-Tracking Spacecraft, *Journal of the Astronautical Sciences* 44, 443-466.
8. Hablani HB (1989), Design of a Payload Pointing Control System for Tracking Moving Objects, *Journal of Guidance, Control, and Dynamics* 12, 365-374.
9. Hablani HB (1994), Multiaxis Tracking and Attitude Control of Flexible Spacecraft with Reaction Jets, *Journal of Guidance, Control, and Dynamics* 17, 831-839.
10. Kalweit CC (1983), Optimum Yaw Motion for Satellites with a Nadir-Pointing Payload, *Journal of Guidance, Control, and Dynamics* 6, 47-52.
11. Shuster MD (1993), A Survey of Attitude Representations, *Journal of the Astronautical Sciences* 41, 439-517.
12. Building GUIs with MATLAB (2018), The Mathworks Company, Inc., Natick, Massachusetts, United States.
13. Vallado DA (2013), *Fundamentals of Astrodynamics and Applications*, Microcosm Press.