Model Order Reduction of Continuous Interval Systems via Time Moment Matching and Pole Clustering Approaches

Dharma Pal Singh Chauhan, V. P. Singh

Abstract**: *In this manuscript, authors propose an algorithm for model order reduction of interval systems. Algorithm utilizes pole clustering and time-moment matching approaches for calculating denominator and numerator of model respectively. In pole clustering approach, poles of the high order system are considered for denominator calculation. According to order of the model, clusters of the poles are framed. Cluster centre of every cluster is determined using inverse distance criterion. Using these cluster centres, model denominator is deduced. Numerator is obtained by equating time moments of system and model. Proposed algorithm is applied on sixth order system and results are compared with other existing techniques which shows that proposed algorithm is superior to other techniques.***

Index Terms**: *Inverse Distance Criterion; Interval System; Model-order Reduction; Pole-Clustering; Time-Moment; Time Moment Matching.***

I. INTRODUCTION

A lot of issues are related with high-order systems, e.g., dimensional problem [1], analysis and design [2], and controller design [3]. So, model order reduction becomes a necessary tool to 4minimize all such problems from high-order systems. In this process, high-order interval system is converted into low-order model. Several methods are available for model order reduction of interval system. Some of them are time-moment matching technique [1, 4], model order reduction of large scale systems [2, 5, 6], method [7], dominant pole retention [8], and pole clustering approach [9]. In time moment matching technique, time-moments of system and model are equated to calculate numerator of the model. Reduced order model using Routh approximants is described in [2, 5, 6] . [8] discusses technique to find poles of the interval system and dominant poles of system are retained in the model. In [9], model denominator is determined using cluster centre of poles.

Proposed technique confirms the stability of the model for stable system due to the reason that the poles of the model are framed from the subset of poles of the system. Also, pole clustering technique is applied on discrete interval system [9] but in the following manuscript, it is applied on continuous interval system.

In proposed reduction technique, pole clustering approach and time moment matching technique are utilized. Reduced order denominator and numerator polynomials are obtained by pole clustering approach and time moment matching technique, respectively. Pole clustering [9] confirms stability of the reduced order model for stable system. Here, authors use an easy formulation for calculating time moments [4] of interval transfer function and formulation is applied on time moment matching technique.

Outline of the manuscript is as follows: Section II contains problem formulation; section III describes main procedure which explains poles calculation, pole clustering, time-moment formulation, and time moment matching technique; section IV demonstrates the procedure with an illustrative example; and section V ends the manuscript with conclusion.

II. PROBLEM FORMULATION

The properties of any system can be explained by it's transfer function. Let, the transfer function of a stable interval system is written as:

$$G(s)=\frac{[a_{n-1}^-,a_{n-1}^+]s^{n-1}+[a_{n-2}^-,a_{n-2}^+]s^{n-2}+\cdots+[a_0^-,a_0^+]}{[b_n^-,b_n^+]s^n+[b_{n-1}^-,b_{n-1}^+]s^{n-1}+\cdots+[b_0^-,b_0^+]} \quad (1).$$

where $\left[a_i^-,a_i^+\right],\forall i\in\{0,1,\dots,n-1\}$ and $\left[b_j^-,b_j^+\right],\forall j\in\{0,1,\dots,n\}$ are the coefficients of system-numerator and system-denominator polynomials respectively; a_i^-,b_j^- and a_i^+,b_j^+ are lower and upper bounds of interval coefficients.

Now, the aim is to find out the reduced order model $\hat{G}(s)$ of degree r where $r<n$ and

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$$\left.\begin{aligned}\hat{G}(s) &= \frac{[\hat{a}_{r-1}^-, \hat{a}_{r-1}^+]s^{r-1} + [\hat{a}_{r-2}^-, \hat{a}_{r-2}^+]s^{r-2} + \cdots + [\hat{a}_0^-, \hat{a}_0^+]}{[\hat{b}_r^-, \hat{b}_r^+]s^r + [b_{r-1}^-, b_{r-1}^+]s^{r-1} + \cdots + [\hat{b}_0^-, \hat{b}_0^+]} \\ &= \frac{\hat{N}_{r-1}(s)}{\hat{D}_r(s)}\end{aligned}\right\} \quad (2)$$

where $\left[\hat{a}_i^-, \hat{a}_i^+\right], \forall i \in \{0,1,\ldots,r-1\}$ and $\left[\hat{b}_j^-, \hat{b}_j^+\right], \forall j \in \{0,1,\ldots,r\}$ are the coefficients of model-numerator and model-denominator polynomials respectively;

$\hat{a}_i^-, \hat{b}_j^-$ and $\hat{a}_i^+, \hat{b}_j^+$ are lower and upper bounds of interval coefficients respectively.

III. MAIN PROCEDURE

Outline for the calculation of the reduced order model is that firstly poles and time-moments of the system are calculated; poles are calculated by the method proposed by Dief [10] and time-moments are calculated by method proposed by Singh et al [4]. Further, according to the model denominator degree, cluster centres of poles are calculated using poles of the system. Furthermore, using these cluster centres denominator of reduced order model is calculated. At last, numerator coefficients of reduced-order model are calculated by using time-moment matching technique as proposed by authors.

1. Calculation of Poles of the System

Poles are calculated by the method proposed by Dief [10] as given below.

Given the system denominator as

$$\begin{aligned}D(s) &= [b_n^-, b_n^+]s^n + [b_{n-1}^-, b_{n-1}^+]s^{n-1} \\ &\quad \ldots + [b_0^-, b_0^+]\end{aligned} \quad (3)$$

(3) can also be written as

$$\begin{aligned}D(s) &= [b_n^-, b_n^+]\left[\begin{aligned}&s^n + \frac{[b_{n-1}^-, b_{n-1}^+]}{[b_n^-, b_n^+]}s^{n-1} + \\ &\cdots + \frac{[b_0^-, b_0^+]}{[b_n^-, b_n^+]}\end{aligned}\right] \\ &= [b_n^-, b_n^+]\left[\begin{aligned}&s^n + [\tilde{b}_{n-1}^-, \tilde{b}_{n-1}^+]s^{n-1} + \\ &\cdots + [\tilde{b}_0^-, \tilde{b}_0^+]\end{aligned}\right].\end{aligned} \quad (4)$$

Let,

$$\left.\begin{aligned}\tilde{D}(s) &= s^n + [\tilde{b}_{n-1}^-, \tilde{b}_{n-1}^+]s^{n-1} + \\ &\quad \cdots + [\tilde{b}_0^-, \tilde{b}_0^+] \\ &\cong s^n + \tilde{b}_{n-1}s^{n-1} + \cdots + \tilde{b}_0\end{aligned}\right\}. \quad (5)$$

Roots of and will be same. Procedure [8, 10] for the calculation of roots is as follows:

State interval matrix of dimension for (5) can be written as,

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\tilde{b}_0 & -\tilde{b}_1 & -\tilde{b}_2 & \cdots & -\tilde{b}_{n-1} \end{bmatrix}. \quad (6)$$

(6)in matrix form can be shown as,

$$A = \left[A_s - \Delta A, A_s + \Delta A\right]. \quad (7)$$

Elements of are formed as,

$$\left.\begin{aligned}a_{s\,i,j} &= \frac{1}{2}\left(a_{i,j}^- + a_{i,j}^+\right), \\ &\forall i, j \in \{1,2,\ldots,n\}\end{aligned}\right\} \quad (8)$$

and elements of ΔA are formed as,

$$\left.\begin{aligned}\Delta a_{i,j} &= \frac{1}{2}\left(a_{i,j}^+ - a_{i,j}^-\right), \\ &\forall i, j \in \{1,2,\ldots,n\}\end{aligned}\right\} \quad (9)$$

Let the roots of state interval matrix A be λ where,

$$\lambda_i = \lambda_i^R = j\lambda_i^I. \quad (1)$$

Here, notations λ_i, λ_i^R, and λ_i^I stand for i^{th} root, real part of i^{th} root, and imaginary part of i^{th} root of state interval matrix A.

$$\left.\begin{aligned}\lambda_i^R(A) &= \begin{bmatrix}\lambda_i^R(A_s - \Delta A \circ X^i), \\ \lambda_i^R(A_s + \Delta A \circ X^i)\end{bmatrix} \\ \lambda_i^I(A) &= \begin{bmatrix}\lambda_i^I(A_s - \Delta A \circ Y^i), \\ \lambda_i^R(A_s + \Delta A \circ Y^i)\end{bmatrix}\end{aligned}\right\}, \quad (2)$$

where, $i \in \{1,2,\cdots,n\}$,and $\circ$ denotes element wise multiplication simultaneously X^i and Y^i denotes eigenvector and inverse eigenvector of i^{th} eigenvalue of state interval matrix A. where,

$$\left.\begin{aligned}X_{m,n}^i &= \operatorname{sgn}\left(y_{R,m}^i x_{R,n}^i + y_{I,m}^i x_{I,n}^i\right) \\ Y_{m,n}^i &= \operatorname{sgn}\left(y_{R,m}^i x_{I,n}^i - y_{I,m}^i x_{R,n}^i\right)\end{aligned}\right\} \quad (3)$$

x and y are elements of eigenvector and reciprocal eigenvector.

2. Pole Clustering Technique

Let, there is a polynomial of degree; it has poles. Now, suppose there is demand of making cluster centres. According to the pole clustering methodology, first of all, clusters from poles are selected. Let, be the number of ways, of selecting clusters from poles, then

$$S = {}^nC_r \times (r!) \times (n-r)^r \quad (13)$$

After selecting r clusters from n poles; cluster centre of every cluster is calculated as follows,
suppose a cluster C_i, where $i \in \{1,2,...,r\}$, is selected. Let, it contains m poles. *i.e.* $p_1, p_2,..., p_m$. Now the cluster centre C_i^c for i^{th} cluster can be calculated from the formula as given below,

$$C_i^c = \left\{ \begin{array}{l} \left\{ \left(\sum_{j=1}^{m} \left(\frac{1}{p_j} \right) \right) \div m \right\}^{-1}, \\ \forall i = \{1,2,...,r\} \end{array} \right\} \quad (4)$$

.

Further, using these cluster centres i.e. $C_1^c, C_2^c,..., C_r^c$; the denominator of the reduced order model can be written as,

$$\hat{D}_r(s) = (s - C_1^c) \times (s - C_2^c) \times \cdots \times (s - C_r^c) \quad (5)$$

3. Determination of Reduced Order Model Numerator

After calculating reduced order denominator, model numerator coefficients are determined by time-moment matching technique [4]. time-moments of system and model are equated for calculating unknown numerator coefficients. In this manuscript authors use the expression for calculating time-moments of interval transfer function derived by Singh et al [4]. Algorithm for calculating unknown numerator coefficients is as follows,
System transfer function (1) can be written as,

$$\left. \begin{array}{l} G(s) = \dfrac{[a_{n-1}^-, a_{n-1}^+]s^{n-1} + [a_{n-2}^-, a_{n-2}^+]s^{n-2} + \cdots + [a_0^-, a_0^+]}{[b_n^-, b_n^+]s^n + [b_{n-1}^-, b_{n-1}^+]s^{n-1} + \cdots + [b_0^-, b_0^+]} \\ = \dfrac{a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + ... + a_0}{b_n s^n + b_{n-1}s^{n-1} + ... + b_0} \\ = T_0 + T_1 s + T_2 s^2 + ... + T_i s^i + ...|_{s=0} \\ \forall i \in \{0,1,2,...,r,...\infty\} \end{array} \right\}. \quad (6)$$

where, T_i stands for $(i+1)^{th}$ time moment of $G(s)$, and also 3^{rd} expression of (6) is called expansion of $G(s)$ around $s = 0$. Simultaneously,

$$\left. \begin{array}{l} T_0 = \dfrac{a_0}{b_0} \\ T_1 = \dfrac{a_1}{b_0} - \dfrac{b_1 T_0}{b_0} \\ \vdots \quad \vdots \quad \vdots \end{array} \right\} \quad (7)$$

in general

$$T_m = \frac{a_m}{b_0} - \sum_{i=0}^{m-1} \frac{b_{m-i} T_i}{b_0} \quad (8)$$

Applying same algorithm for model, i.e

$$\left. \begin{array}{l} \hat{G}(s) = \dfrac{[\hat{a}_{r-1}^-, \hat{a}_{r-1}^+]s^{r-1} + [\hat{a}_{r-2}^-, \hat{a}_{r-2}^+]s^{r-2} + \cdots + [\hat{a}_0^-, \hat{a}_0^+]}{[\hat{b}_r^-, \hat{b}_r^+]s^r + [\hat{b}_{r-1}^-, \hat{b}_{r-1}^+]s^{r-1} + \cdots + [\hat{b}_0^-, \hat{b}_0^+]} \\ = \dfrac{\hat{a}_{r-1}s^{r-1} + \hat{a}_{r-2}s^{r-2} + ... + \hat{a}_0}{\hat{b}_r s^r + \hat{b}_{r-1}s^{r-1} + ... + \hat{b}_0} \\ = \hat{T}_0 + \hat{T}_1 s + \hat{T}_2 s^2 + ... + \hat{T}_i s^i + ...|_{s=0} \\ \forall i \in \{0,1,2,...,r,...\infty\} \end{array} \right\} \quad (9)$$

in general

$$\hat{T}_m = \frac{\hat{a}_m}{\hat{b}_0} - \sum_{i=0}^{m-1} \frac{\hat{b}_{m-i} \hat{T}_i}{\hat{b}_0}. \quad (10)$$

Further, numerator coefficients of reduced order model are calculated by matching time moments of system and model i.e.,

$$\left. \begin{array}{l} \hat{T}_0 = T_0 \\ \hat{T}_1 = T_1 \\ \vdots \quad \vdots \quad \vdots \\ \hat{T}_{r-1} = T_{r-1} \end{array} \right\} \quad (11)$$

Furthermore, numerator can be written as,

$$\hat{N}_{r-1}(s) = \hat{a}_0 + \hat{a}_1 s + \cdots + \hat{a}_{r-1}s^{r-1} \quad (12)$$

Finally, from (5) and (12) reduced order model can be written as,

$$\hat{G}(s) = \frac{\hat{N}_{r-1}(s)}{\hat{D}_r(s)} \quad (13)$$

IV. EXAMPLE

Consider a sixth-order system i.e.,

$$G_6(s) = \frac{[2,3]s^5 + [70,71]s^4 + [762,763]s^3 + [3610,3611]s^2 + [7700,7701]s + [6000,6001]}{[1,2]s^6 + [42,42]s^5 + [571,572]s^4 + [3491,3492]s^3 + [10060,10061]s^2 + [13100,13101]s + [6000,6001]} \quad (14)$$

From **Error! Reference source not found.** to(3), poles of (14) can be calculated as,

$$\left. \begin{array}{l} p_1 = [-20.021, -15.835] \\ p_2 = [-20.021, -1.9426] \\ p_3 = [-1.9731, -1.0506] \\ p_4 = [-1.0506, -0.5424] \\ p_5 = [-0.5424, -0.3188] \\ p_6 = [-0.31876, -0.24872] \end{array} \right\} \quad (15)$$

Further, selecting $C_1(p_1, p_2, p_3)$ and $C_2(p_3, p_4, p_5)$ as clusters, applying (4) cluster centres can be written as,

$$\left.\begin{array}{l} C_1^c = [-4.9447, -1.9611] \\ C_2^c = [-0.5057, -0.3333] \end{array}\right\} \quad (16)$$

Further, using (7) and (16) reduced order denominator can be written as,

$$\hat{D}_2(s) = s^2 + [2.2944, 5.4509]s + [0.6536, 2.5006] \quad (17)$$

After calculating reduced-order denominator $\hat{D}_2(s)$, first two time-moments of the system (14) are calculated, using (8) as given below,

$$\left.\begin{array}{l} T_0 = [0.9998, 1.0002] \\ T_1 = [-0.9007, -0.8991] \end{array}\right\} \quad (18)$$

Applying (11) and (12) reduced order denominator is written as follows,

$$\hat{N}_1(s) = [0.0166, 7.4421]s + [0.6535, 2.5010] \quad (19)$$

Finally, using(17), (19) and (13) reduced order model written as given below,

$$\hat{G}_2(s) = \frac{[0.0166, 7.4421]s + [0.6535, 2.5010]}{s^2 + [2.2944, 5.4509]s + [0.6536, 2.5006]} \quad (20)$$

Model Proposed by Sastry et al[6]

$$\hat{G}_2^s(s) = \frac{[2, 2]s + [8.9365, 8.9496]}{s^2 + [6.5103, 6.5114]s + [8.9454, 8.9478]} \quad (21)$$

Model proposed by Bandhyopadhyay et al[1]

$$\hat{G}_2^b(s) = \frac{[-8308.1, 320100]s + [3801.4, 9471.8]}{[3684.7, 13384]s^2 + [553.34, 52957]s + [3802, 9470.2]} \quad (22)$$

Model proposed by Singh et al[4]

$$\hat{G}_2^{sh}(s) = \frac{[-3734.3, 3563.9]s + [1478.6, 4031.7]}{[3684.7, 13384]s^2 + [553.34, 52957]s + [3802, 9470.2]} \quad (23)$$

Step responses of system and model plotted some of the step responses are given in figure 1.

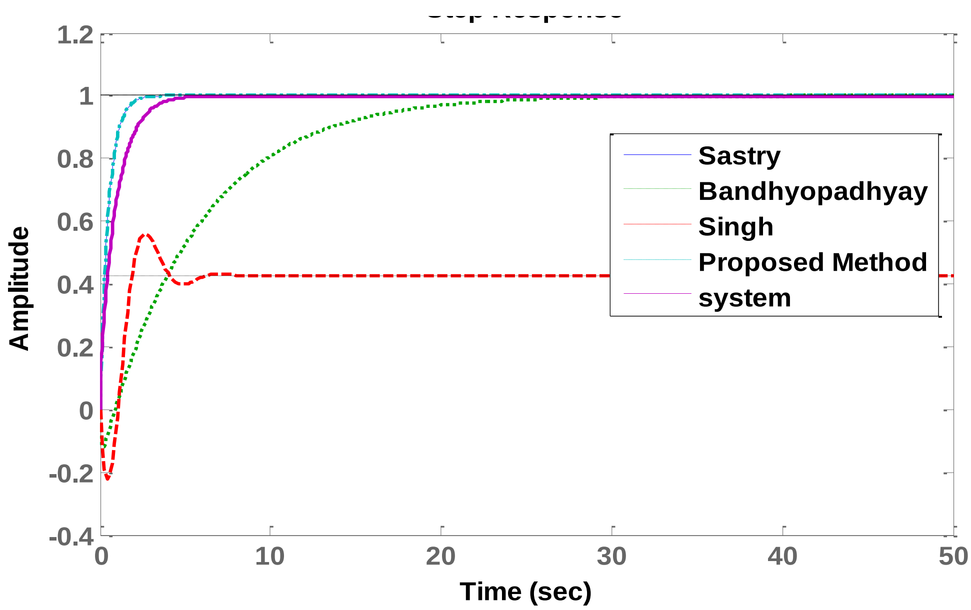

Figure 1: Step Responses of System and Models

According to the figure 1, step response of reduced order model by proposed method has better transient and steady state response on system as compared to other existing techniques i.e. Sastry et al. [2], Singh et al. [4] and Bandhyopadhyay et al. [1].

V.CONCLUSION

A method for model order reduction of continuous interval system is described. In proposed method, denominator of reduced order model is obtained by pole clustering approach, this insures stability of the model for stable system. Further, authors utilize an expression for calculating time-moments of system, and numerator of reduced order model is calculated via time moment matching technique. Finally, proposed model reduction technique is applied on a sixth-order interval system and simultaneously using Kharitonov theorem results are compared with other existing technique.

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