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On Degree Dominating Functions in Graphs

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***Abstract:** A set S the number of vertices in a graph G is said to be a dominating set if every vertex in $V - S$ is adjacent to some vertex in S. A degree dominating function (DDF) is a function $f: V(G) \mid \{0,1,2,3,\dots,\Delta(G)+1\}$ having the property that every vertex v of S is assigned with $deg(v)+1$ and all remaining vertices with zero. The weight of a degree dominating function f is defined by $w(f) = \sum_{v\in S}(deg(v)+1)$. The degree domination number, denoted by $\gamma_{deg}(G)$ is the minimum weight of all possible DDFs. In this paper, we extended the study of the degree domination number of some classes of graphs.*

***Keywords:** Domination Number, Degree Domination, Mycielski Graph, Pentagonal chain. AMS Subject Classification: 05C69, 05C76, 68R10.*

Abbreviations:
DDF: Degree Dominating Function

I. INTRODUCTION

In this paper, we consider all graphs to be simple, finite, non-trivial, and undirected. Let G be a graph with a vertex set $V(G)$ and edge set $E(G)$. For basic graph theory terminologies not explicitly defined in this paper, please refer to [1]. One of the fast-developing fields in graph theory is the study of domination and its related concepts. A dominating set S The number of vertices in a graph G is a subset such that every vertex $v \in V - S$ is adjacent to at least one vertex in S. Domination is a fundamental concept used as an active tool in graph theory. The initiation of a new parameter in domination in graphs, called "degree domination number," was introduced by N.C. Demirpolat and E. Kilic [2]. The domination number, denoted by $\gamma(G)$ of graph G is the minimum cardinality of a dominating set of G. Gwas introduced by T.W. Haynes and S.T. Hedetniemi [4]..

Definition 1.1. The degree dominating function (DDF) is a function $f: V(G) \to \{0,1,2,\dots,\triangle(G)+1\}$ having the property that every vertex v of the dominating set S is assigned with deg(v)+1 and all remaining vertices with zero. The weight of a degree dominating function f is defined by $w(f) = \sum_{v\in S}(\deg(v)+1)$.

The degree domination number, denoted by $\gamma_{\deg}$, is the minimum weight of all possible DDFs.

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Definition 1.2. (Mycielski Graph) [3]

Consider a graph G with $V(G) = \{v_1, v_2, \dots, v_n\}$. Apply the following steps to the graph. G :
(i) Take the set of new vertices $U = \{u_1, u_2, \dots, u_n\}$ and add edges from each vertex $u_i \in U$ to the vertices v_j if the corresponding vertex v_i is adjacent to v_j in G.

(ii) Take another new vertex w_0 and add edges joining each element in U.

Here, the new graph obtained is the Mycielski Graph, denoted by $\mu(G)$ of the graph G.

Theorem 1.3. Let P_n be a path graph with $n \geq 4$. Then

$$\gamma_{\deg}(\mu(P_n)) = \begin{cases} \frac{8n+3}{3}, & \text{if} \quad n \equiv 0(\text{mod}3) \\ \frac{8n+2}{3}, & \text{if} \quad n \equiv 2(\text{mod}3) \\ \frac{8n+7}{3}, & \text{if} \quad n \equiv 1(\text{mod}3) \end{cases}$$

Proof: Let $V(\mu(P_n)) = V(P_n) \cup X \cup Y$, where $V(P_n) = \{v_1, v_2, \dots, v_n\}, X = \{u_1, u_2, \dots, u_n\}$, and $Y = \{x_0\}$. It is known that the maximum degree of the graph. $\mu(P_n)$ is $\triangle(G) = n$. also, there are three types of degrees: (i) vertices of degree 4, (ii) vertices of degree 3, and (iii) vertices of degree n.

Then we define $f: V(\mu(P_n)) \to \{0,1,2,\dots,n+1\}$.
If $n \equiv 0(\text{mod}3)$, then $f(v_{3i-1}) = 5$ for $1 \leq i \leq \frac{n}{3}$ and $f(x_0) = n$
If $n \equiv 2(\text{mod}3)$, then $f(v_{3i-1}) = 5$ for $1 \leq i \leq \frac{n-2}{3}$ and $f(v_n) = 3$ and $f(x_0) = n$,
If $n \equiv 1(\text{mod}3)$, then $f(v_{3i-1}) = 5, f(v_n) = 3$ and $f(x_0) = n$,
For all remaining vertices $f(v) = 0$. It is easy to generalize that. f is the degree domination function of $\mu(P_n)$ of weight
$5.\frac{n}{3} + n + 1 = \frac{5n+3n+3}{3} = \frac{8n+3}{3}$, if $n \equiv 0(\text{mod}3)$,
$5.\frac{n-2}{3} + 3 + n + 1 = \frac{8n+2}{3}$, if $n \equiv 2(\text{mod}3)$, and
$5.\frac{n-1}{3} + 3 + n + 1 = \frac{8n+7}{3}$, if $n \equiv 1(\text{mod}3)$.
Thus,

$$\gamma_{\deg}(\mu(P_n)) = \begin{cases} \frac{8n+3}{3}, & \text{if} \quad n \equiv 0(\text{mod}3) \\ \frac{8n+2}{3}, & \text{if} \quad n \equiv 2(\text{mod}3) \\ \frac{8n+7}{3}, & \text{if} \quad n \equiv 1(\text{mod}3) \end{cases}$$

Theorem 1.4. For $n \geqslant 3, \gamma_{\deg}(\mu(C_n)) =$

$$\begin{cases} \frac{8n+3}{3}, & \text{if} \quad n \equiv 0(\text{mod}3) \\ \frac{8n+8}{3}, & \text{if} \quad n \equiv 2(\text{mod}3) \\ \frac{8n+13}{3}, & \text{if} \quad n \equiv 1(\text{mod}3) \end{cases}$$

Proof: Let $V(\mu(C_n)) = V(C_n) \cup X \cup Y$, where $V(C_n) = \{v_1, v_2, \dots, v_n\}, X = \{u_1, u_2, \dots, u_n\}$, and $Y = \{x_0\}$. It is known that there are three types of degrees: (i) vertices of degree 4, (ii) vertices of degree 3, and (iii)vertices of degree n.

Then we define $f: V(\mu(C_n)) \to \{0,1,2,\ldots,n+1\}$.

If $n \equiv 0(\mathrm{mod}3)$, then $f(v_{3i-1}) = 5$, for all $1 \le i \le \frac{n}{3}$ and $f(x_0) = n$.

If $n \equiv 2(\mathrm{mod}3)$, then $f(v_{3i-1}) = 5$, for all $1 \le i \le \frac{n-2}{3}$, $f(v_n) = 3$ and $f(x_0) = n$.

If $n \equiv 1(\mathrm{mod}3)$, then $f(v_{3i-1}) = 5$, for all $1 \le i \le \frac{n-1}{3}$, $f(v_n) = 3$ and $f(x_0) = n$,

For all remaining vertices $f(v) = 0$. It is easy to generalize that. f is the degree domination function of $\mu(C_n)$ of weight $5.\frac{n}{3} + 5 + n + 1 = \frac{5n+3n+3}{3} = \frac{8n+3}{3}$, if $n \equiv 0(\mathrm{mod}3)$,

$5.\frac{n-2}{3} + n + 1 = \frac{8n+8}{3}$, if $n \equiv 2(\mathrm{mod}3)$,

and $5.\frac{n-1}{3} + 5 + n + 1 = \frac{8n+13}{3}$, if $n \equiv 1(\mathrm{mod}3)$.

Thus,

$$\gamma_{\deg}(\mu(C_n)) = \begin{cases} \dfrac{8n+3}{3}, & \text{if} \quad n \equiv 0(\mathrm{mod}3) \\ \dfrac{8n+8}{3}, & \text{if} \quad n \equiv 2(\mathrm{mod}3) \\ \dfrac{8n+13}{3}, & \text{if} \quad n \equiv 1(\mathrm{mod}3) \end{cases}$$

II. WINDMILL GRAPH

The Windmill graph $W_{(m,n)}$ is the graph obtained by taking n copies of the complete graph K_m sharing a common vertex.

Theorem 1.5. Let $G \cong W_{(m,n)}$ be a Windmill graph, where $m \geqslant 2, n \geqslant 1$.

$\gamma_{\deg}(W_{(m,n)}) = nm - n + 1$

Proof: Let $G \cong W_{(m,n)}$ be a Windmill graph on $(m-1)n + 1$ vertices and $\frac{mn(m-1)}{2}$ edges, where $m \geqslant 2, n \geqslant 1$, Windmill graph $W_{(2,n)}$ It is also called the star graph. $K_{(1,n)}$ and Windmill graph $W_{3,n}$ It is also called a friendship graph. F_n. Let $\deg(v) = \triangle$. Then $S\{v\}$ is a minimum dominating set of $W_{(m,n)}$ and maximum degree $\triangle(G) = n(m-1)$. By the definition of DDF, $f: V(G) \to \{0,1,2,\ldots,\triangle+1\}$ and the Degree domination function must consist of a vertex $\{\deg(v)+1\}$. Hence, the degree domination number is

$$\gamma_{\deg}(G) = \sum_{v \in s} f(v) = n(m-1) + 1$$

Thus,

$$\gamma_{\deg}(W_{m,n}) = n(m-1) + 1, \text{where } m \geqslant 2, n \geqslant 1.$$

Theorem 1.6. : Let $G \cong B_n$ be a book graph, for $n \geqslant 1$. Then $\gamma_{\deg}(B_n) = 2n + 4$.

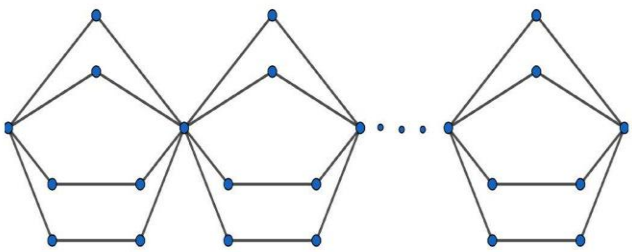
Proof: Let $B_n = S_{n+1} \times P_2$ be a book graph with $2n+2$ vertices, $V = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n, x, y\}$. In B_n graph: there are two vertices x and y Which form the dominating set with the neighbours $N(x) = \{v_1, v_2, \ldots, v_n, y\}$ and $N(y) = \{u_1, u_2, \ldots, u_n, x\}$. It is seen that the set. $S = \{x, y\}$ is the minimum dominating set of B_n. The maximum degree of the graph G is $\triangle(G) = n + 1$. By the definition of the Degree domination function, $f: V(B_n) \to \{0,1,2,\ldots,\triangle+1\}$ and the Degree domination function must consist of vertices $\{\deg(x)+1, \deg(y)+1\}$.

Hence, the degree domination number is

$$\gamma_{deg}(G) = \sum_{v \in s} f(v)$$
$$\gamma_{deg}(G) = ((n+1)+1) + ((n+1)+1)$$
$$\gamma_{deg}(G) = 2n + 4$$

III. PENTAGONAL CHAIN

Pentagonal chains are essential tools in network theory and some chemical applications. Pentagonal double chain [5] consisting of k units of pentagons are denoted by $C^2_{(5,k)}$ and illustrated in Figure 1:



[Fig.1: Pentagonal Double Chains $C^2_{5,k}$]

The order of $C^2_{5,k}$ is $n = 7k + 1$ and the size of $C^2_{5,k}$ is $m = 10k$.

Theorem 1.7. : Let $G \cong C^2_{(5,k)}$ be a pentagonal double chain. Then, $\gamma_{\deg}(C^2_{(5,k)}) = 10 + (k-1)$, where $k \geq 1$ is the number of pentagons.

Proof: Let $V(C^2_{5,k}) = X \cup Y \cup M$, where X is the set of vertices of degree 4, Y is the set of vertices of degree 8 and M It is the set of vertices of degree 2. It is easy to see that $X \cup Y$ is a minimum dominating set of G and Let $S = \{v_1, v_2, \ldots, v_k, v_{(k+1)}\}$ be a minimum dominating set of G, where $\{v_1, v_{k+1}\}$ A set of vertices of degree 4. If $k = 1$, then $S_1 = \{v_1, v_2\}$ is a minimum dominating set of G. The degree of each vertex of S_1 is 4 . By the definition of DDF, $f: V(G) \to \{0,1,2,3,4,5\}$ and the DDF must consist of vertices $\{\deg(v_1)+1, \deg(v_2)+1\}$. Hence, the degree domination number is

$$\gamma_{\deg}(G) = \sum_{v \in S_1} f(v) = (4+1) + (4+1)$$
$$\gamma_{\deg}(G) = 5 + 5 = 10 + (1-1)$$
$$\gamma_{\deg}(G) = 10 + (k-1) \text{ for } k = 1$$

If $k = 2$, then $S_2 = \{v_1, v_2, v_3\}$ is a minimum dominating set of G. The vertices of degree v_1 and v_3 is 4_4 and the vertex of degree v_2 is 8 . By the definition of DDF, $f: V(G) \to \{0,1,2,3,\ldots,9\}$ and the DDF must consist of vertices $\{\deg(v_1)+1, \deg(v_2)+1, \deg(v_3)+1\}$.

Hence, the degree domination number is

$$\gamma_{\deg}(G) = \sum_{v \in S_2} f(v) = 5 + 9 + 5 = 10 + 9$$
$$\gamma_{\deg}(G) = 10 + 9(2-1)$$
$$\gamma_{\deg}(G) = 10 + 9(k-1), \text{for } k = 2$$

Continuing this process, for any $k \geqslant 1$, we have

$\gamma_{\deg}(G) = 5 + 9 + 9 + 9 + \cdots + 9 + 5 = 10 + 9(k-1)$.

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IV. FIRECRACKER GRAPH

A Firecracker graph $F(m,n)$ It is a graph obtained by a series of interconnected nodes. m copies of n stars by linking one leaf from each.

Theorem 1.8. For any Firecracker graph $F(m,n), \gamma_{\deg}\left(F_{(m,n)}\right) = mn$, where $n \geq 2$ Proof: Let $G \cong F(m,n)$ be a Firecracker graph on mn vertices with $(mn) - 1$ edges and let S be a minimum dominating set of a graph G. The Firecracker graph is obtained from a series of interconnected elements. m copies of n stars by linking one leaf from each. For each of the n stars, we choose all the central vertices $x_1, x_2, \dots, x_m$ as one set, $S = \{x_1, x_2, \dots, x_m\}$. So, we will get a minimum dominating set, therefore $\gamma(G) = m$. The maximum degree of the graph G is $\Delta(G) = n - 1$. By the definition of $DDFf: V(G) \to \{0,1,2,\dots,\triangle+1\}$ and the DDF must consist of vertices $\{\deg(x_1) + 1, \deg(x_2) + 1, \dots, \deg(x_m) + 1\}$.

Hence, the degree domination number is

$$\gamma_{\deg}(G) = \sum_{v \in s} f(v) = [(n-1)+1] + [(n-1)+1] + \cdots + [(n-1)+1]$$

$$\gamma_{\deg}(G) = n + n + \cdots + n \text{ (mtimes)}$$

$$\gamma_{\deg}(G) = mn$$

V. CONCLUSION

A. Fan Graph

A Fan graph $F_{m,n} = \overline{K_m} + P_n$, where $\overline{K_m}$ is a disconnected graph and P_n is the path on n vertices.

Theorem 1.9. For any fan graph $F_{(m,2)}$

Proof: Let $G \cong F_{(m,2)}$ be a fan graph on $m + 2$ vertices with $2m + 1$ edges. Let S be a minimum dominating set of a graph G. of the Fan graph, the $G = \overline{K_m} + P_2$. Let $V(P_2) = \{u, v\}$, there are two vertices available; if we choose any one vertex from the path P_2, then all the other vertices of G Our chosen vertex dominates them. Hence, the minimum dominating set S of G is $\{u\}$ or $\{v\}$. The maximum degree of the graph G is $\triangle(G) = m + 1$. By the definition of DDF $f: V(G) \to \{0,1,2,\dots,\triangle+1\}$ and the DDF must consist of vertices $\{\deg(u) + 1\}$.

Hence, the degree domination number is,

$$\gamma_{\deg}(G) = \sum_{v \in S} f(v) \gamma_{\deg}(G) = (m+1) + 1 = m + 2$$

DECLARATION STATEMENT

After aggregating input from all authors, I must verify the accuracy of the following information as the article's author.

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